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# Quelques recherches sur les raies faibles dans les spectres optiques

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*ABSTRACT*. En plus de la faiblesse absolue des raies, c'est surtout leur faiblesse relative qui interdit leur observation lorsqu'elles sont trop voisines d'autres raies. Elles sont alors masquées par la lumière provenant de l'étalement des raies voisines, qui est dû essentiellement à la diffraction dans le spectrographe.

Il est souvent possible d'observer les raies faibles en modifiant dans la source elle-même les rapports d'intensité entre les différentes raies, grâce au phénomène d'autoabsorption, dont quelques exemples d'utilisation sont donnés.

On peut aussi modifier profondément la figure de diffraction classique de faron à en atténuer les 'pieds'. Cette 'apodisation' est obtenue, soit par des diaphragmes de formes diverses, soit par des écrams absorbants dégradés suivant des lois convenables. Quelques indications sont données sur le calcul de ces lois, sur la réalisation des écrans, et sur les résultats obtenus, consistant en des améliorations de contraste au voisinage des raies, dans un rapport atteignant facilement 10 000.

Le plus interessant des résultats spectroscopiques obtenus jusqu'à présent est la mise en évidence et l'étude des raies d'intercombinaison de l'hélium. La plus facile à observer de ces raies se présente comme un satellite de la raie jaune, distant de 1 A., et 10 000 fois plus faible. Il a même été possible d'observer l'effet Zeeman de cette raie; cet effet Zeeman est d'ailleurs différent de celui que l'on doit attendre, et cette contradiction pose un problème de rayonnement qui n'a pas reçu de solution.

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I SHOULD like to say first how happy I am to be the fifth Holweck Prizewinner; it is for me a very great honour to have been recognized by the distinguished scientists who form the Physical Society Council.

Knowing the important part played by this prize in the friendly relations between scientists of our two countries, I am very proud to be the object of one of these expressions of friendship, the idea of which we owe to the eminent Foreign Secretary of the Physical Society. Professor Andrade's initiative will keep green for many years the memory of the life and death of Fernand Holweck, who was one of the most original and creative of our French physicists.

I knew Holweck a little. I admired his intuition, the acuteness of his way of thinking, and his experimental skill. I know too that his judgment of those of his colleagues whose work lacked character was sometimes a bit severe. I appreciate all the more the honour that is done to me today, because his name is attached to the Prize.

I am also very proud to see my name appear on the Holweck Prize list, which opened so brilliantly with those of Professor Andrade, of my old friend Charles Sadron, and of Professor Bates and Professor Rocard. My personal merits are rather modest, when compared with those of the former names, but it is impossible to suggest in public that there might be any doubt about the wisdom of the decisions of the Physical Society Council!

I have preferred to say a few words in English. I am sure now you will have no regret if I change language. You will surely understand me far better if I avoid going on torturing the language of Newton and Faraday.

Le sujet de ma conférence est une brève revue de ce que l'on doit faire, et de ce que l'on peut trouver lorsqu'on veut, par une curiosité peut-être excessive, chercher des raies spectrales situées à quelques dixièmes d'Angstrom d'autres raies quelques milliers de fois plus intenses qu'elles.

Les interféromètres se prêtent mal à de telles recherches. Cependant si les différences de longueur d'ondes deviennent très faibles ils sont les seuls appareils utilisables. Il y a en Angleterre d'éminents spécialistes de l'interférométrie qui savent combien on est alors limité par les questions de contraste. Kuhn a déjà obtenu dans ce sens des résultats intéressants par l'emploi de deux appareils Fabry-Perot en série, et Dufour (1949) a étudié le problème dans sa thèse récente. Mais je ne considérerai aujourd'hui que de l'emploi des spectrographes à prismes ou à réseau, c'est-à-dire ceux dans lesquels on forme l'image monochromatique d'une fente.

## §1. LIMITE DE RÉSOLUTION POUR DEUX RAIES D'INTENSITÉS TRÈS DIFFÉRENTES

Si une raie faible, ou un groupe de raies faibles, était isolée dans le spectre à étudier il y aurait bien des moyens de la détecter malgré sa faiblesse: emploi de sources plus intenses, en particulier de sources longues, poses de grande durée dans le cas de la détection photographique, systèmes à grande constante de temps dans le cas de la détection photoélectrique. De toute façon, plus la raie est faible, plus il faut de moyens puissants, et de temps, pour l'étudier, mais si cette faiblesse absolue des raies exige beaucoup de travail, elle ne constitue pas un obstacle insurmontable.

Malheureusement les raies ne sont jamais seules, et les raies ' faibles ' encore moins que les autres. Elles peuvent être mélées à toutes sortes de lumières parasites, en particulier à des spectres d'impuretés, et tous les spectroscopistes connaissent bien les difficiles problèmes de purification que cela pose. Mais la recherche des raies faibles devient vraiment très difficile quand il se trouve dans leur voisinage des raies beaucoup plus intenses qu'elles, et qui appartiennent normalement au même spectre. Si la différence de longueur d'onde $\Delta\lambda$ des deux raies est trop faible, ou leur rapport d'intensité $K$ trop grand, il devient impossible de détecter la raie faible, et si $K$ est par exemple de l'ordre de 1000, $\Delta\lambda$ peut être 10 ou 100 fois plus grand que la limite de résolution conventionnelle $\delta\lambda$ définie pour deux raies d'égale intensité. Cela est dû au fait que la lumière s'étale autour de la position géométrique d'une raie même parfaitement monochromatique, beaucoup plus qu'on ne le croit d'habitude. Si l'on trace la courbe de répartition de l'éclairement dans l'image monochromatique d'une fente, cette courbe se compose d'une partie centrale ou ' corps ', et d' ' ailes ' ou ' pieds ' beaucoup plus faibles, dont seule une réprésentation en ordonnées logarithmiques peut donner une idée correcte. On réserve le plus souvent le nom d'ailes aux parties de l'image relativement intenses et proches du corps de la raie, et le nom de pieds aux parties de faible intensité relative—par exemple inférieure à $5 \times 10^{-3}$ du maximum et qui peuvent s'étendre très loin. Les aberrations géométriques provoquent généralement un élargissement du corps de l'image, et l'appartition d'ailes, souvent dissymétriques. Mais les pieds sont dûs le plus souvent à la plus essentielle des limitations de la définition, qui est la diffraction. J'ai souvenir qu'on n'enseigna que la figure de diffraction d'une fente donnée par un objectif rectangulaire de largeur $a$ à la distance $f$ était composée d'une frange centrale, de largeur $\lambda f/a$, et de franges latérales (figure 1) * de largeur deux fois plus petite, et d' ' intensité négligeable '. Mais la dixième frange a encore une intensité relative de $10^{-3}$ (figure 2), ce qui est loin d'être négligeable quand les raies à séparer ont un contraste de $10^4$, et l'intensité des franges décroit seulement comme l'inverse du carré de leur numéro. Un calcul très simple montre alors que le pouvoir de résolution pour deux raies de contraste $K$ serait ainsi, pour $K$ grand, environ $\sqrt{K/\pi}$ fois plus faible que pour deux raies d'égale intensité, en supposant que la raie faible est résolue si son intensité est au moins égale à celle du pied de la raie forte à sa place.

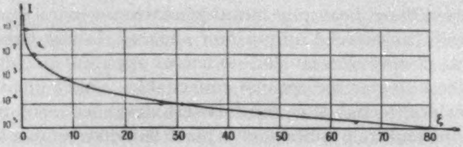

Figure 2. Loi de décroissance de l'éclairement de franges en fonction de leur numéro; ordonnées logarithmiques.

Ces résultats simples sont vrais seulement si c'est la diffraction qui régit entièrement la formation des images: tels est le cas si, préférant la résolution à la luminosité, on adopte une distance focal $f$ suffisament grande pour que le 'grain' de la plaque, les largeurs de fente utilisées ou l'étendue des taches d'aberration soient au plus égales à la largeur $\lambda f/a$ de la tache centrale de diffraction. On dit alors que l'ouverture $a/f$ est égale à *l'ouverture résolvante*, qui est de l'ordre de 1/40 en général. Il en est bien ainsi dans les grands spectrographes à haut pouvoir de résolution utilisés pour étudier la structure des spectres d'émission atomiques et moléculaires: on dispose alors de tout le pouvoir de ***résolution intrinsèque*** du système dispersif.

Mais de tels appareils sont peu lumineux et on préfère souvent employer une ouverture $m$ fois plus grande que l'ouverture résolvante: dans la cas des appareils photographiques on gagne alors en luminosité comme $m^2$, et l'appareil possède alors un pouvoir de résolution réel $m$ fois plus petit que le pouvoir de résolution intrinsèque.† Dans la cas de deux raies de contraste $K$, la relation est alors moins simple à établir entre le pouvoir de résolution $R_k$ et le contraste $K$, mais une étude plus détaillée montre que $R_k$ est $\sqrt{K/\pi}\sqrt{m}$ fois plus faible que le pouvoir de résolution $R$ réellement obtenu avec deux raies d'égale intensité. On peut donc dire que dans ce cas la diffraction joue relativement moins que dans la cas d'un spectrographe peu ouvert, mais elle joue cependant encore beaucoup trop.

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* All photographs are reproduced on Plates I–IV, facing page 976.

† Le même résultat peut être obtenu dans le cas des appareils photoélectriques en agissant sur les largeurs de fente: le pouvoir de résolution obtenu est alors inversement proportionnel à la luminosité (Jacquinot et Dufour 1948).

Il semblerait donc que pour augmenter $R_k$ on soit, de toute façon, obligé d'augmenter $R$. Mais si l'appareil travaille déjà avec son pouvoir de résolution intrinsèque $R_0$, il n'y a pas d'autre moyen que d'augmenter les dimensions de l'appareil dispersif ou sa dispersion angulaire, ce qui est souvent impossible. Si, au contraire, l'appareil est assez ouvert pour que son pouvoir de résolution réel $R$ soit inférieur à sa valeur intrinsèque $R_0$, on peut augmenter $R$ en diminuant l'ouverture, c'est à dire en choisissant une longueur focale $m$ fois plus grande : on gagne alors un facteur $\sqrt{m}$ sur $R_k$, mais en revanche on perd un facteur $m^2$ sur la luminosité, ce qui est très désavantageux. Un autre procédé consisterait à faire comme dans les monochromateurs doubles, ou l'on a surtout en vue l'élimination de la lumière *diffusee*, beaucoup moins génante en général que les ' pieds ' de diffraction : après la traversée d'un premier appareil, la raie faible est isolée au moyen d'une fente et renvoyée dans un deuxième appareil, qui, dans ce cas, peut être le même grâce à un système optique convenable. Avec une telle disposition l'intensité relative des 'pieds' de la raie forte est élevée au carré, mais on ne peut ainsi déceler qu'une seule raie faible dont la place doit être connue à l'avance, et, en outre, la double traversée de l'appareil fait perdre beaucoup de lumière.

C'est à deux autres procédés que j'ai eu recours à Bellevue pour les recherches que j'y ai entreprises avec la concours de quelques collaborateurs dont les principaux sont J. Brochard, et M$^{\text{lle}}$ Dossier. Le premier de ces procédés, qui n'est pas toujours applicable, consiste à modifier, dans la source lumineuse elle-même les rapports d'intensités des raies. Le second, plus général, consiste à modifier la figure de diffraction de façon à en atténuer les pieds : c'est ce que nous avons appelé 'l'apodisation'.

## § 2. MODIFICATION DES RAPPORTS D'INTENSITÉ PAR AUTOABSORPTION

On peut, dans une source lumineuse modifier soit les rapports d'intensité vrais des raies, soit les rapports de leurs intensités apparentes. De nombreux exemples de modification d'intensités vraies sont connus, et c'est ainsi que de nombreuses raies interdites ont pu être étudiées en favorisant l'accumulation d'atomes sur des niveaux de départ plus ou moins métastables. Et je ne veux pas parler ici des raies forcées, ou l'on agit directement sur les probabilités de transition de raies qui doivent normalement ne pas exister. Mais il arrive souvent que l'on ne peut aucunement agir sur les populations relatives des différents niveaux, ni sur les probabilités de transition, mais où l'on peut agir sur les intensités relatives *apparentes* des raies émises par une source donnée. Cette action est possible grâce au phénomène d'autoabsorption des raies, souvent considéré comme génant, sinon négligeable, et dont, le plus souvent, on omet prudemment de parler. Toute raie émise par un point d'une source est, en effet, partiellement absorbée dans la source même sur la parcours de la lumière et l'intensité apparente résulte de l'intégration combinée de l'émission et de l'absorption. Or les coefficients d'absorption sont dans quelques cas proportionnels aux coefficients d'émission (loi de Kirchhoff), et presque toujours, sans leur être aussi simplement liés, croissent en même temps qu'eux. Il en résulte que les raies intenses sont relativement beaucoup plus autoabsorbées que les raies faibles, ce qui provoque une diminution du contraste entre les intensités apparentes des raies (Jacquinot 1941). Cet effet est d'autant plus sensible que les coefficients d'absorption sont plus grands, et que la source a une épaisseur plus grande dans le sens de la visée ; cela conduit a employer des sources assez fortement excitées, et, surtout ayant la forme de tubes très longs. Dans un tube long, tout se passe donc comme si les raies intenses, donc fortement absorbées, n'étaient émises que par une tranche très mince à la sortie du tube, alors que les raies faibles non absorbées sont émises par le tube entier et ont une intensité proportionnelle à sa longueur. La calcul est très simple à faire pour des raies supposées monochromatiques (ou de 'profil rectangulaire'). Si $L$ est la longueur du tube, $I_0L$ l'intensité qu'aurait une raie si l'absorption était nulle, et $a$ le coefficient d'absorption, l'intensité apparente est $I=(I_0/a)(1-e^{-aL})$, qui se réduit à $I_0L$ si $a$ est très faible et à $I_0/a$ si $L$ est grand. Le contraste entre une raie faible et une raie forte se trouve donc divisé par un facteur $G=aL$, qui est le gain dû à l'autoabsorption.

Le même résultat est encore valable en chaque région d'une raie non monochromatique dont la largeur serait nettement plus grande que la limite de résolution du spectrographe. Mais pour une raie réelle dont la largeur est plus faible que cette limite, le calcul doit comporter une intégration sur toute la raie, ce qui nécessite une connaissance exacte de la *forme réelle* de la raie, c'est à dire de la loi suivant laquelle varient les coefficients d'émission et d'absorption en fonction de la longueur d'onde. Or on ne peut se contenter de faire des hypothèses sur cette forme de raie, par exemple d'admettre que la répartition d'intensité est due seulement à l'effet Doppler, car, si l'autoabsorption est forte, même des intensités très faibles à grande distance du centre de la raie peuvent fausser les résultats. De toute façon bien que le calcul soit souvent impossible, le gain $G$ de contraste dû à l'autoabsorption, est moins grand que celui que l'on pourrait calculer pour une raie monochromatique.

Les gains que l'on obtient peuvent être considérables. C'est ainsi qu'avec des tubes de 2 mètres de longueur, on obtient pour certaines raies de l'hydrogène ou de l'hélium des diminutions de contraste dans un rapport atteignant 100. Si l'on observe dans un tel tube la raie rouge H$\alpha$ de l'hydrogène, la raie du deuterium apparait seulement 50 fois plus faible, bien que la teneur en deuterium ne soit que de 1/5 000.

Malheureusement cette méthode d'autoabsorption n'est pas très générale : toutes les raies ne sont pas aussi facilement absorbables, et l'emploi de sources très longues est très difficile pour produire les spectres des éléments non gazeux aux températures moyennes. En outre il est souvent très difficile de connaître dans quel rapport ont été modifiées les différentes intensités relatives, et s'il est ainsi possible de mettre en évidence des raies nouvelles, la mesure de leurs intensités relatives vraies peut être très compliquée, et quelquefois impossible.

C'est en partie grâce à cette méthode d'autoabsorption que nous avons pu à Bellevue poursuivre de nombreuses études sur des raies interdites de l'hélium et de l'hydrogène. L'étude de l'autoabsorption sera, en outre, poursuivie pour elle-même, car elle permet d'obtenir de nombreux renseignements sur les populations des niveaux et les probabilités de transition : les mesures existantes actuellement à ce sujet sont en effet souvent discordantes et il arrive de voir deux auteurs différents donner d'un même coefficient d'absorption deux valeurs dans le rapport de 1 à 50.

Si l'on ne peut modifier les intensités relatives apparentes des raies, ou si cette modification est insuffisante, il faut s'attaquer au phénomène de diffraction lui-même. Là encore nous distinguerons deux méthodes, dont l'une utilise une fente réduite à un point et l'autre une fente ordinaire.

## §3. APODISATION PAR ÉCRANS GÉOMETRIQUES

Les astronomes, qui travaillent souvent avec des sources ponctuelles, connaissent bien la méthode qui permet de voir, par exemple, le compagnon de Sirius. On remplace le contour circulaire de l'objectif par un contour polygonal : les anneaux de diffraction sont alors remplacés par une figure étoilée, et l'éclairement décroit, entre les branches, plus rapidement que dans le cas des anneaux. Cette méthode peut être transposée à la spectroscopie, en utilisant une fente réduite à un point et un contour d'objectif polygonal tourné de telle sorte que l'axe de dispersion soit bissectrice entre deux branches de la figure de diffraction étoilée. C'est cette méthode que nous avons, avec A. Couder, proposée en 1939 (Couder et Jacquinot 1939), en employant un contour carré dont les cotés font un angle de 45° avec l'arête du prisme. La figure de diffraction est celle de la figure 3 : l'éclairement y décroît, suivant les branches de la croix comme $1/x^2$ mais, suivant les diagonales, comme $1/x^4$. L'inconvénient du procédé est qu'on ne peut utiliser qu'une fente, sinon ' ponctuelle ', du moins très courte, dont la longueur ne dépasse pas de 5 à 50 fois sa largeur suivant la distance de la raie faible cherchée. Cette limitation de la longueur de la fente est très génante, car une véritable raie se voit mieux qu'un point, et la mesure des distances est plus difficile. En outre la longueur de la fente, qui n'intervient pas dans la luminosité ' photographique ', intervient directement dans le cas de l'exploration photoélectrique directe du spectre.

Cependant l'emploi de cette méthode permet d'obtenir très vite des résultats nouveaux : la figure 4 montre l'exemple de raies très faibles au voisinage de la raie bleue 4471 A. de l'hélium, et qui ont été littéralement ***devoilées***, car sans le diaphragme les branches de la croix auraient été rabattues sur elles.

On peut encore accroître l'efficacité de la méthode et mieux dégager dans une direction les abords du centre de la figure de diffraction en utilisant des contours de formes mieux étudiées. Le calcul complet de la figure de diffraction d'un diaphragme de forme quelconque est assez laborieux, mais il se simplifie si l'on considère des types tels que ceux de la figure 5 et si l'on ne s'intéresse qu'a l'éclairement sur l'axe de symétrie O$\xi$, parallèle à O$x$, de la figure de diffraction : l'éclairement s'obtient alors par une intégrale simple de Fourier appliquée à la fonction $y=f(x)$ entre $a$ et $b$. J'indiquerai plus loin comment on peut déterminer $f(x)$ pour que la figure de diffraction présente les qualités voulues : je me contenterai seulement ici de montrer quelques figures de diffraction dans lesquelles la lumière diffractée a été déplacée de la région intéressante beaucoup mieux qu'avec un simple contour carré ou polygonal (figures 6, 7, 8).

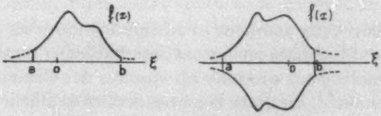

Figure 5. Contours de diaphragmes.

Evidemment ces diaphragmes ' géométriques ' présentent toujours, pour la spectroscopie, l'inconvénient de nécessiter une fente très courte, mais ils peuvent avoir d'autres applications. La figure 9 montre, en particulier, leur application à l'étude de la lumière diffractée par un défaut d'homogénéité du verre (' fil ' perpendiculaire à O$x$), étude faite par M. Françon, de l'Institut d'Optique.

## §4. APODISATION PAR ÉCRANS ABSORBANTS

Pour modifier la figure de diffraction tout en employant une fente comme source il faut agir, non plus sur le contour de l'objectif, ou plus généralement de la pupille de sortie du spectroscope, mais sur la répartition des amplitudes sur cette pupille. Ces répartitions sont obtenues en disposant sur la pupille un écran absorbant dont la transmission varie d'un point à un autre. En particulier si l'amplitude lumineuse, au lieu de s'annuler brusquement à la limite de la pupille, s'annule progressivement, toute périodicité disparaît dans la figure de diffraction. Mais ce qui est intéressant n'est pas de supprimer le périodicité, c'est d'obtenir des éclairements relatifs aussi faibles que possible tout près du centre de l'image: tel est le problème de l'‘apodisation’.

Je ne développerai pas le traitement mathématique de la question, exposé en détail par ailleurs (Dossier, Boughon et Jacquinot 1950), mais je veux seulement indiquer pourquoi il n'est pas simple. Le cas le plus simple, et aussi le plus utile pour le spectroscopiste, est celui d'une pupille de contour rectangulaire dont un axe, $Oy$, est parallèle a la fente, l'amplitude lumineuse $f(x)$ étant constante le long de toute parallèle à $Oy$; l'éclairement $I(\xi)$ dans la figure de diffraction alors donnée par le carré de la transformée de Fourier de $f(x)$, fonction nulle en dehors de la pupille. Si, réciproquement, on se donne arbitrairement $I(\xi)$ on trouvera une ‘fonction pupillaire’ $f(x)$, qui, généralement, ne sera pas nulle en dehors de la pupille. Le problème réciproque n'a donc, en général, aucune solution. On doit se contenter d'imposer à la ‘ fonction d'image ’ $I(\xi)$ certaines conditions de concentration ou de décroissance de l'énergie dites ‘critères d'apodisation’: les équations intégrales auxquelles on aboutit ainsi ne peuvent être résolues que par des méthodes d'approximation. La méthode adoptée consiste à exprimer $f(x)$ suivant un polynome de Fourier à deux ou trois termes et à en déterminer les coefficients de façon à satisfaire au critère d'apodisation.

Suivant les critères choisis et la largeur d'image dans laquelle on cherche à concentrer l'énergie on aboutit à des fonctions différentes donnant des images plus ou moins apodisées, dont la partie centrale ou ‘ corps ’ est plus ou moins élargie. En général, plus on veut une apodisation prononcée, plus il faut consentir à élargir le corps de l'image et à perdre de la lumière. Il n'existe pas *une* solution meilleure que les autres, mais des solutions différentes suivant le problème posé, recherche d'un satellite faible et lointain, ou très faible et proche. A titre d'exemple, les caractéristiques d'une solution moyenne sont données dans la figure 10 et dans la tableau, comparées à celle de la pupille de dimensions identiques et de transmission uniforme: cette comparaison a été établie en tenant compte de la largeur finie de la fente, choisie dans chaque cas de façon à assurer le meilleur compromis entre la luminosité et le pouvoir de résolution.

Caracteristiques d'un écran apodiseur.

| | | | | | | |
|---|---|---|---|---|---|---|
| Loi de transmission : | $f(x)=0,4935+0,4865\cos\pi x+0,0200\cos 2\pi x$ (ecran $C_2$) | | | | | |
| Perte de pouvoir résolvant : | 30% | | | | | |
| Perte de luminosité théorique : | 50% | | | | | |
| Coefficient d'apodisation $K(\xi)$ | $\xi$ | 2,5 | 4,5 | 6,5 | 9,5 | $+\infty$ |
| | $K(\xi)$ | 270 | 5,550 | 830 | 482 | 336 |

$\xi$ est le rapport de l'abscisse à la limite de résolution conventionnelle.

Le coefficient d'apodisation $K(\xi)$ est le rapport, en un point $\xi$ donné, de l'éclairement relatif sans écran à l'éclairement relatif avec écran.

On voit que, bien que l'apodisation soit considérable, la perte de pouvoir résolvant au sens conventionnel est seulement de 30 % et la perte de lumière de 50 %.

La figure 11 montre les figures de diffraction comparées d'une ouverture rectangulaire avec et sans écran apodisant. Les cotés du rectangle sont dans le rapport de 1 à 5, les lignes d'égale opacité de l'écran apodiseur étant parallèles aux petits cotés ($Oy$). Les franges de diffraction suivant $Ox$ sont supprimées, alors que les autres, 5 fois plus larges sont inchangées. L'élargissement du corps de l'image, visible sur les franges verticales, est ici de l'ordre de 1,7.

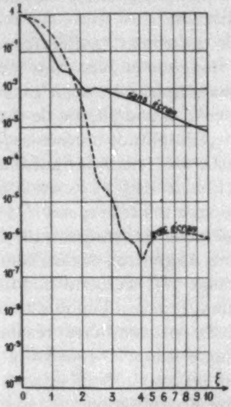

Figure 10. Loi de décroissance de l'éclairement avec l'écran apodiseur $C_2$ (voir tableau), et une largeur de fente source égale à la largeur d'une frange de diffraction.

Des coefficients d'apodisation de même ordre de grandeur sont obtenus dans le cas des appareils d'ouverture plus grande que l'ouverture résolvante: on peut donc obtenir une plus forte amélioration de contraste, et au prix d'une perte de lumière beaucoup plus faible qu'en diminuant l'ouverture jusqu'a sa valeur resolvante. Dans ce cas, d'ailleurs, l'apodisation n'est accompagnée d'aucune diminution du pouvoir résolvant, puisque celui ci est fixé uniquement par la limite de résolution de la plaque photographique.

Ces écrans apodisants peuvent être réalisés par dépôts de métaux évaporés dans le vide. La variation d'épaisseur suivant la loi désirée s'obtient par déplacement de l'écran, pendant le dépôt, derrière un cache de forme convenablement calculée.

Ce déplacement doit s'effectuer d'un mouvement alternatif uniforme, ce qui entraine certaines complications mécaniques: un appareil (figures 12 et 13) a été construit spécialement pour la préparation de ces écrans. La préparation d'écrans ayant la symétrie de révolution serait beaucoup plus simple, mais les écrans de révolution sont moins bien adaptés au cas du spectrographe.

## § 5. DEFAUTS DE L'APPAREIL DISPERSIF

Les différentes méthodes d'apodisation n'agissent que sur le phénomène de diffraction et elles perdent toute efficacité si le système optique possède des défauts

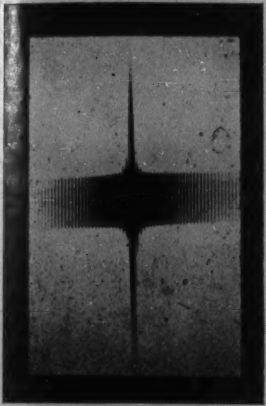

Figure 1. Photographie des franges de diffraction d'une fente.

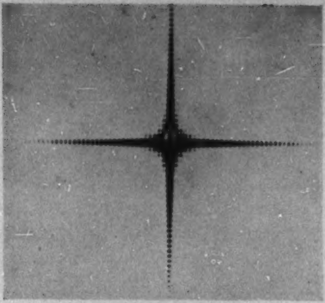

Figure 3. Figure de diffraction d'un point par une ouverture carrée.

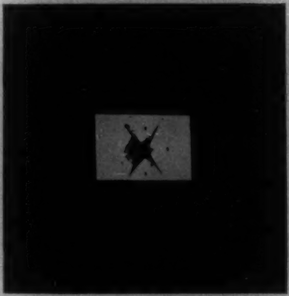

Figure 4. Raies satellites de la raie He 4471, qui apparaissent grâce à l'utilisation d'un diaphragme carré tourné de 45° par rapport à la direction de dispersion.

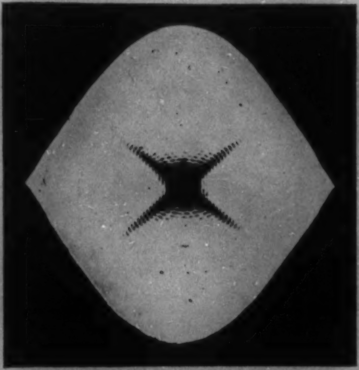

Figure 6. Figure de diffraction d'un point donnée par un diaphragme de contour $f(x)=\cos \frac{1}{2}\pi x$. Le contour de diaphragme a été dessiné de façon a encadrer la figure de diffraction.

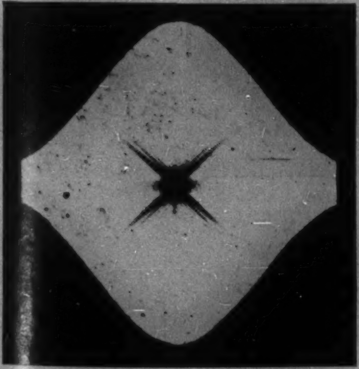

Figure 7. Même figure que 6 avec $f(x)=\exp(-2x^2)$.

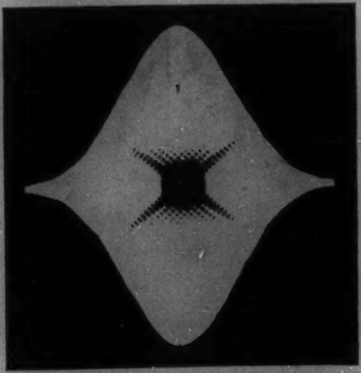

Figure 8. Même figure que 6 avec $f(x)=0{,}4935+0{,}4865\cos \pi x+0{,}0200\cos 2\pi x$.

PLATE I.

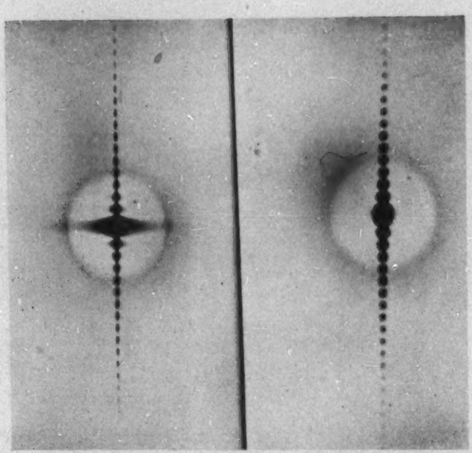

Figure 11. Image d'un point à travers un écran rectangulaire uniforme (à gauche) et à travers un même écran apodisant dans une direction (à droite).

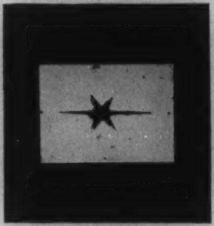

Figure 15. Pieds 'spectraux' de la raie 4471 de l'hélium, qu'on peut supprimer en abaissant la pression.

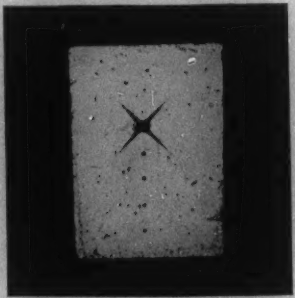

Figure 16. La raie jaune 5875 et la raie d'intercombinaison qu'on distingue juste entre les bras de la croix, à droite.

produisant des pieds d'intensité comparable à ceux de la figure normale de diffraction: ces défauts passent généralement inaperçus car on ne les sépare pas de la diffraction normale. Un bon moyen de les mettre en évidence est de faire une photographie très surexposée avec une fente de très faible hauteur et l'un des diaphragmes géométriques décrits précédemment. Ce contrôle a été appliqué à un réseau concave de Rowland de 21 pieds et le résultat en est montré sur la figure 14 obtenue avec le raie verte du mercure dans le premier ordre avec un diaphragme en forme de losange. La lumière parasite, presque aussi génante que la lumière diffractée qui est ainsi mise en évidence, est due aux irrégularités des traits du réseau, et il n'y a aucun moyen de la diminuer; l'effet est encore plus important dans les ordres plus élevés. Ce réseau est donc impropre aux recherches sur les satellites faibles.

Je n'ai pas fait d'essais sur d'autres réseaux, car, heureusement, j'ai eu à ma disposition un rémarquable instrument à prisme, construit en 1935, sur la demande de M. Cotton, par A. Couder. C'est un gros prisme à liquide, associé, en montage autocollimateur, à une lentille de 9 mètres de distance focale. La section droite du faisceau sortant du prisme à 20 cm. de diamètre et l'ensemble a, dans le bleu, un pouvoir résolvant comparable à celui d'un grand réseau de Rowland dans le deuxième ordre, sans en avoir aucun des inconvénients: la figure 16 montre une raie très fortement surexposée (environ 10 000 fois), donnant une lumière parasite très faible en comparaison de la lumière diffractée. C'est grâce à cet instrument, probablement unique, que j'ai pu obtenir la plupart de mes résultats sur les raies faibles.

## §6. EXEMPLE DES RAIES D'INTERCOMBINAISONS DE L'HÉLIUM

Tous ces procédés, qui permettent l'observation de raies faibles au voisinage de raies intenses peuvent être appliqués à de nombreux problèmes, tels que la recherche des raies d'isotopes rares, l'étude de la forme des raies loin de leur centre, là ou l'intensité due à l'effet Doppler est négligeable devant l'intensité, même très faible, due aux autres causes d'élargissement; on peut aussi, chemin faisant, avoir l'heureuse chance de rencontrer des raies nouvelles auxquelles on ne songeait pas.

Et, pour en étudier l'exemple le plus intéressant, je vais raconter l'histoire de la raie jaune de l'hélium.

J'ai commencé à étudier cette raie en 1939: c'etait alors pour rechercher un isotope de masse 5 dont on pouvait penser, à cette époque, qu'il était stable quoique très peu abondant. Le développement de toutes les techniques qui viennent d'être exposées a son origine dans cette recherche.

J'étudiai donc quelques raies de l'hélium avec un tube de 20 cm. de longueur et le grand prisme muni d'un diaphragme carré à 45°, qui était le seul procédé que je connaisse alors. En surexposant les photographies j'obtins tout d'abord des étoiles à 6 branches (figure 15) au lieu des étoiles à 4 branches que j'attendais: c'est que les raies présentaient elle mêmes des pieds dûs non pas à la diffraction, mais au phénomène même de l'émission. Ces pieds, très faibles, échappent aux observations habituelles où on peut les confondre avec la lumière diffractée car ils décroissent suivant la même loi. Il suffit de diminuer suffisamment la pression dans le tube pour faire disparaître ces pieds ' spectraux '. Les abords de la raie se trouvent alors suffisamment dégagés pour que l'observation de raies faibles y soit possible: les figures 16 et 4 relatives aux raies 5875 $(2\,^3P-3\,^3D)$ et 4471 $(2\,^3P-4\,^3D)$ montrent les raies qui apparaissent alors et dont l'intensité est de l'ordre du dixmillième de celle de la raie qu'elles accompagnent.

Parmi toutes les raies qui apparaissent ainsi (Jacquinot 1939), certaines sont des raies ‘ forcées ’ dont l’intensité relative augmente avec la densité de courant, et sont dues aux champs interioniques : il a été possible par la suite, avec des moyens plus puissants—tubes plus poussés et utilisation de l’autoabsorption—d’en étudier complètement l’effet Zeeman (Brochard et Jacquinot 1945), et cette étude à conduit à des résultats curieux sur la répartition des champs électriques interioniques qui régnent à l’intérieur d’une décharge placée dans un champ magnétique.

D’autres raies ont une intensité relative *vraie* indépendante des conditions d’excitation : tel est le cas de celle qui accompagne à 1,12 A. de distance la raie jaune 5875 ($2^3P$–$3\,^3D$), de celle qui accompagne à 1,92 A. de distance la raie rouge 6 678 ($2\,^1P$–$3\,^1D$) et de leurs homologues supérieures dans ces deux séries. Malgré leur faible intensité relative vraie ces raies s’observent bien, même sans dispositif apodiseur grâce à l’autoabsorption, avec un tube de 2 mètres, ainsi que le montre l’enregistrement de la figure 17 obtenu directement avec un dispositif photoélectrique sensible placé dans le plan focal du grand spectrographe à prisme. Leur probabilité de transition relative a pu être mesurée exactement par M. Brochard (1949) en tenant compte de l’effet d’autoabsorption. On indentifie facilement ces raies comme des intercombinaisons entre niveaux de singulets et niveaux de triplets conformément à la figure 18. On ne connaisait jusqu’alors que deux ou trois raies d’intercombinaison, situées dans l’extrême ultraviolet : encore ces raies étaient elles contestées à cause de la démonstration, donnée par Heisenberg, de l’impossibilité de ces combinaisons si l’on ne tient pas compte du spin. Cependant ces raies ne sont pas complètement interdites, à cause des interactions spin–orbite et comme la transition a lieu entre deux niveaux de partités différentes, la règles des parités de Laporte permet leur existence en qualité de raies dipolaires électriques : par contre cette même règle les interdit rigoureusement pour le rayonnement quadrupolaire électrique.

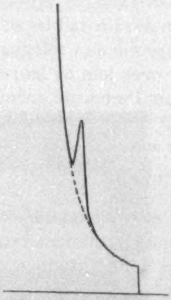

Figure 17. Enregistrement direct de la raie rouge d’intercombinaison de l’hélium.

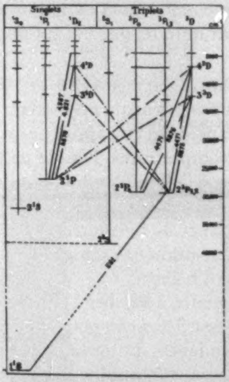

Figure 18. Schéma du spectre de l’hélium, montrant les nouvelles raies d’intercombinaison.

L’étude de l’effet Zeeman, à condition de faire à la fois des observations transversales et *longitudinales*, doit permettre de savoir à quel type de rayonnement elles correspondent. Mais, pour ces raies qui sont déjà difficiles à observer, l'étude de l'effet Zeeman est à limite des possibilités ; c'est pourquoi il n'a pu être observé que sur la raie jaune (Brochard et Jacquinot 1946) . Les clichés obtenus sont cependant suffisamment nets pour que l'on puisse affirmer qu'il s'agit d'une raie quadrupolaire, contrairement à ce que l'on devait attendre.

Cette raie voile donc la règle des parités, qui, par sa nature même doit être rigoureuse et n'est violée, je crois, par aucune autre raie connue. La raison de cette anomalie n'est pas encore apparue, et aucune explication théorique ne semble possible actuellement. Il existe des raies homologues, dans des atomes plus lourds, telle la raie 5770 $(2\,^1P{-}3\,^3D_2)$ du mercure, dont l'intensité relative est très grande ; mais cette raie, dont l'effet Zeeman est bien connu, est dipolaire et satisfait à la règle des parités. Nos recherches tendent actuellement à rendre possible l'étude de l'effet Zeeman des autres raies d'intercombinaison du type P–D de l'hélium, et des mêmes raies dans les éléments suivants à deux électrons optiques. Ces études présenteront sans doute de grandes difficultés, ne serait ce que parce que les raies d'intercombinaison homologues de celles de l'hélium ne sont pas encore connues dans les éléments qui suivent l'hélium, tel le beryllium. C'est afin de les rendre possibles que nous avons poursuivi l'étude de l'apodisation par écrans absorbants ; ces écrans, à cause de nombreuses difficultés de réalisation n'ont pas encore été utilisés sur le grand spectrogrpahe de Bellevue, mais ils le seront très prochainement. En outre le spectrographe est maintenant équipé d'un système à double faisceau, pour l'enregistrement direct des raies par voie photoélectrique, qui permet d'étudier même des raies extrêmement faibles.

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# Electron Microscopic Studies on Aqueous Sols

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*Communicated by G. I. Finch; MS. received 21st August* 1950

*ABSTRACT.* Electron micrographs are shown for aqueous sols containing the following colloidal suspensions: graphite, stearic acid, vanadium pentoxide, and sulphur. The results obtained explain certain divergences noticed during the measurement of depolarization of light scattering in these colloidal solutions.

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## § 1. INTRODUCTION

It is generally assumed that colloidal graphite in aqueous sols consists of laminated anisotropic particles. Subrahmanya *et al.* (1944) have shown that colloidal stearic acid also consists of disc-shaped particles similar to colloidal graphite. In a recent study on the effect of an electric field on the degree of depolarization of light scattering in colloidal solutions, Subrahmanya and Rao (1949) have shown that the variation of the depolarization factor for the vertically and horizontally polarized light on the application of an electric field indicates a similarity between colloidal graphite and colloidal stearic acid. But with unpolarized light the two sols behave differently, showing thereby that the shape of the particles in the two cases may not be similar. The stearic acid in this case is similar to vanadium pentoxide sol, which consists of elongated particles. The twinkling of the colloidal particles and the schlieren experiments can only indicate the anisotropy of the particles but not the shape. An electron-microscopic study of these sols has therefore been undertaken with a view to determining the shape of the colloidal particles.

## § 2. EXPERIMENTAL

The preparation of colloidal graphite, stearic acid and vanadium pentoxide has been described in a previous paper (Subrahmanya and Rao 1949). Colloidal sulphur was prepared by the oxidation of aqueous hydrogen sulphide with sulphur dioxide. All these had to be suitably diluted before evaporating on the grid.

Of the several materials tried for the supporting film, 'Formvar' proved to be the best. The 'Formvar' membrane was removed on to the grid and dried. A drop of the colloidal solution was then placed on the membrane and evaporated slowly. Rapid evaporation had to be avoided to prevent the coagulation of the colloid. After drying, the particles were gold-shadowed at grazing incidence from a distance of 6 cm. from the gold source.

The electron microscope used in this investigation was constructed in Professor G. I. Finch's laboratory in the Imperial College of Science and Technology, London. It is a three-stage instrument worked at 80 to 100 kv. with a resolving power of about 200 A.

## § 3. RESULTS

Figure 1* indicates a typical electron micrograph for colloidal graphite. This consists of two micrographs taken for different samples of the colloid. The sample

* For all Figures see Plate.

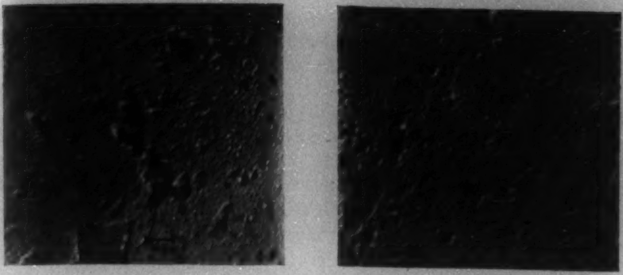

Figure 1. Colloidal graphite shadow cast with gold. × 5000.

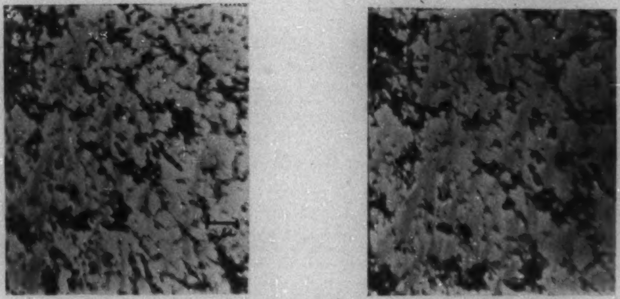

Figure 2. Collodial stearic acid shadow cast with gold. × 5000.

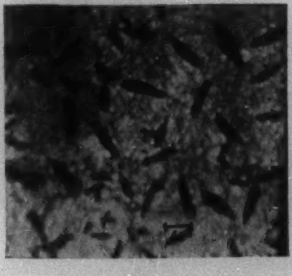

Figure 3. Colloidal vanadium pentoxide. × 5000.

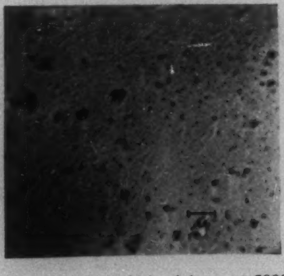

Figure 4. Colloidal sulphur. × 5000.

consists of particles of varying sizes. An ultra-microscopic determination of the size of the particles gave a value of $0{\cdot}138\,\mu$ for the average diameter. The point of interest that has to be specially noted in this figure is the small length of the shadow cast by the particles. This indicates definitely that the colloidal graphite is laminated and that the thickness of the lamina is a small fraction of the average diameter.

Figure 2 is the electron micrograph for stearic acid. Unlike the graphite, the particles of stearic acid seem to have all kinds of shapes. The average diameter as determined by the ultra-microscope is $0{\cdot}236\,\mu$, i.e. about twice that of colloidal graphite. The shadows cast by the particles in this case are much longer than in the previous case. This indicates that the thickness of the particles is much greater than that of the colloidal graphite. The electron microscopic study therefore clearly indicates that there is a vast difference in shape between colloidal graphite and colloidal stearic acid. In fact stearic acid seems to be a mixture of elongated and laminated particles, coming midway between graphite and vanadium pentoxide (Figure 3). The divergence in shape between graphite and stearic acid is responsible for the difference in behaviour of the two sols when the degree of depolarization is measured under the influence of the electric field (Subrahmanya and Rao 1949).

Figure 3 was obtained from freshly prepared vanadium pentoxide. It is interesting to note the presence of the two types of particles. The larger ones are about $1\,\mu$ in length and $0{\cdot}3\,\mu$ in diameter, while the fine structure exhibits a thread-like appearance. On standing, the sol gives only the fine fibrous structure, as has been noticed by Huber and Zbinden (1949). This indicates that the large particles of the freshly prepared vanadium pentoxide undergo spontaneous disintegration on keeping.

A micrograph or colloidal sulphur is shown in Figure 4. This picture is similar to the electron micrograph of colloidal gold. The particles appear to be more or less spherical and very much smaller in diameter. Colloidal sulphur does not indicate any change in the degree of depolarization with orientation of the particles.

The coagulation of the particles during the evaporation of the sol on the 'Formvar' membrane may next be considered. The average diameter for the particle size obtained by the electron microscopic method is close to the value obtained with the ultra-microscope. This indicates that very little coagulation takes place when the sol is evaporated on the 'Formvar' membrane. The high dilution of the sol, the like charges on the colloids, and the presence of a stabilizing agent go a long way towards inhibiting coagulation.

## ACKNOWLEDGMENTS

The author is deeply grateful to Professor G. I. Finch for valuable guidance and for providing laboratory facilities. His thanks are also due to Mr. M. Bluhm for help in using the electron microscope.

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# Photoconductivity in Magnesium Antimonide Layers

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*MS. received* 17*th February* 1950, *and in amended form* 1*st May* 1950

*ABSTRACT.* Layers of $Mg_3Sb_2$ evaporated *in vacuo* are found to exhibit semiconducting properties, although both constituents are normally metals.

Under suitable conditions the layers are photosensitive. Measurements have been carried out on the spectral distribution of sensitivity, and show bands with long wavelength edges at $1{\cdot}5\,\mu$, $2{\cdot}6\,\mu$ and $3{\cdot}5\,\mu$. The former is identified with intrinsic conductivity, and the two latter with impurity levels.

Activation energies found from resistance–temperature measurements are very similar to the optical activation energies corresponding to the above wavelengths.

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## § 1. INTRODUCTION

Until recently only two classes of semiconductor have been recognized, namely: (i) elements, (ii) compounds having as anion a member of group VI of the periodic table, i.e. oxides—tellurides.

A previous paper (Moss 1950) has shown that some compounds having a member of group V as the anion should have semiconducting properties, and that, as a result of the very large polarizabilities of these anions, such compounds should have interesting photoconductive properties. For example, the $As^{---}$ ion has twice the polarizability of the $Te^{--}$ ion (Pauling 1927a) with the same ionic radius (Pauling 1927b).

In order to produce photosensitive films of such compounds by vacuum evaporation methods, it is desirable that the vapour pressures of the component elements should be roughly equal, so that if some degree of dissociation of the compound occurs the components will evaporate at similar rates and will condense in nearly stoichiometric proportions. From this point of view the most suitable element to use in combination with arsenic is cadmium, and the properties of $Cd_3As_2$ layers are described by Moss (1950).

The present paper describes the extension of the work to cover antimonides. As regards vapour pressure, the best element to combine with antimony is magnesium (Landolt–Bornstein 1936). Also from the phase diagram of the Mg–Sb system a definite compound $Mg_3Sb_2$ seems to be formed (Hansen 1936). Accordingly the preparation and investigation of this compound was undertaken.

Soon after the work was started a paper by the Russian workers Boltaks and Zhuze (1948) appeared, giving measurements on the electrical properties of this compound, and verifying that it is indeed a semiconductor.

## § 2. PREPARATION OF BULK MATERIAL AND PHOTOSENSITIVE LAYERS

To form the compound, accurately stoichiometric proportions of 'Specpure' materials were baked together in a sealed-off evacuated quartz tube (pressure $\sim 10^{-6}$ mm. Hg on ionization gauge) at 1,050° c. for $2\frac{1}{2}$ hours.

All cells were made of Pyrex glass in the form of small Dewar flasks. Electrodes of either graphite or platinum about 5 mm. wide with 1 mm. separation were painted on the bottom of the inner part of the Dewar flask.

Early attempts to form layers by evaporation in an oven at temperatures below the softening point of Pyrex (i.e. ~ 600° c.) always resulted in layers with metallic properties. Experiments were therefore carried out with evaporators built inside the cells. Small quartz crucibles of 1·5 mm. diameter and 3 mm. length were made to hold the $Mg_3Sb_2$. For a typical layer 1 to 2 mg. of material were placed in a crucible, which fitted into a tungsten helix placed 1·5 to 2·0 cm. from the electrode surface.

A number of cells were made using various evaporation rates. Two of these had resistances of over $2 \times 10^4$ ohms at room temperature and tens of megohms at liquid air temperature, and showed some decrease in resistance on illumination whilst cool. However, on warming to room temperature the layers lost their semiconducting properties and became metallic in nature. In view of these results, subsequent cells were not in general subjected to such strong cooling.

The next cell produced was slightly photosensitive both at room temperature and at 195° K. Spectral sensitivity measurements carried out at both temperatures showed there was some sensitivity at wavelengths beyond the beginning of the Pyrex absorption region. Accordingly subsequent cells were made with 'bubble' windows of thin Pyrex, such as would give good transmission up to 4·5 $\mu$. Most of these later cells were sensitive, and the detailed measurements carried out on four of them are described.

## § 3. CELLS A AND B

For cell A the layer was formed in high vacuum: the cell was evacuated, the system (including ionization gauge) being pumped and outgassed until the gauge indicated $0{\cdot}8 \times 10^{-6}$ mm. Hg, and the cell then outgassed at 250° c. for an hour. After cooling, the material was evaporated (in about 3 seconds) to form a layer of 85 kΩ resistance. The layer was opaque and had a metallic appearance.

For cell B the layer was evaporated at backing pressure, (approximately $10^{-3}$ mm. Hg), ordinary air being present in the system. A blackish opaque layer resulted, the resistance of which was 30 kΩ.

### 3.1. *Current–Voltage and Signal–Voltage Relations*

A current–voltage curve was measured for cell A to verify that Ohm's law was obeyed. The current and voltage were measured on suitable direct reading meters. The results are plotted in Figure 1 (*a*), from which it is seen that Ohm's law is well obeyed for potentials of up to 70 volts across the layer, corresponding to a field of 700 v/cm. The slight deviation at higher fields is probably due to electrical heating of the layer.

With interrupted radiation falling on the layer, the relation between the A.C. signal produced and the D.C. voltage applied to the layer was determined. It was found to be linear for the same range of voltage as used in the previous experment, as shown in Figure 1 (*a*).

### 3.2. *Response Time*

The response time was measured by observing the time required for the signal due to a square pulse of illumination to decay after removal of the illumination. For cell A the signal decayed to $1/e$ of its initial value in $8 \times 10^{-4}$ second.

### 3.3. *Thermo-electric Power*

For both cells it was found that either junction between layer and electrode, when heated, became positive, showing the current carriers to be electrons.

### 3.4. *Signal–Energy Relation*

The relation between the signal output from the cell and the energy incident on the layer was determined using chopped radiation from a lamp of known power and temperature. The incident energy, calculated by the inverse square law, is shown plotted against the signal in Figure 1 (*b*). For both cells the observed points lie in straight lines of slope unity, showing the signal to be proportional to the incident energy over the range of measurements, i.e. at least up to $3 \times 10^{-3}$ watts.

### 3.5. *Spectral Distribution of Sensitivity*

For these measurements a lithium fluoride monochromator was used with a Nernst source, the radiation being chopped at 80 c/s. The response of the layers would be linear over the whole range of these measurements since the maximum energy used was only 10% of that used in the above signal–energy experiments. Thus the curves obtained as the ratio (cell signal)/(incident energy) are effectively constant energy spectral sensitivity curves.

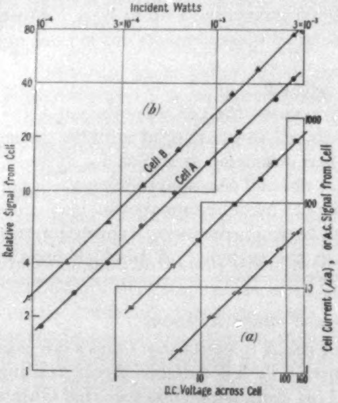

Figure 1.
(*a*) Current/voltage (■) and signal/voltage (<·>) curves. (*b*) Signal/energy relation.

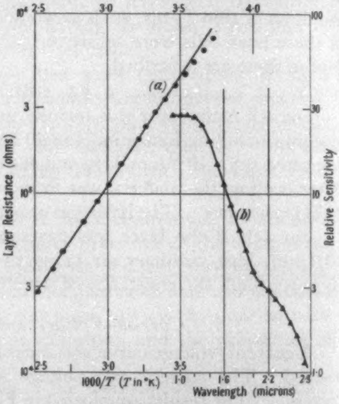

Figure 2.
(*a*) Resistance/temperature. (*b*) Spectral sensitivity.

The results obtained with cell A are shown in Figure 2 (*b*). The curve shows a flat portion from 0·9 μ to 1·2 μ, then an exponential fall in sensitivity up to 2·0 μ after which there is an indication of a further band of sensitivity around 2·6 μ. This latter is presumably some form of impurity band, while the short wavelength band is probably due to intrinsic conductivity. For photoconductive cells there is no absolute threshold wavelength at which the sensitivity suddenly vanishes. The significant part of the spectral sensitivity curve is where the exponential fall commences. As previously (Moss 1949), the 'threshold' is taken where the sensitivity has fallen to half its value on the flat part of the curve, i.e. $\lambda_{1/2} = 1{\cdot}53\,\mu$.

Hall effect measurements have recently been carried out on this material by Boltaks and Zhuze (1948) from which the width of the forbidden zone is quoted as 0·6 to 0·8 ev. The figure of 0·8 ev. (which is preferred since the slope of the Hall constant–temperature curve may be decreased by impurities) is equivalent to a wavelength of just over 1·5 $\mu$, in close agreement with the 'threshold' wavelength for the above cell.

The fact that the 2·6 $\mu$ band appears in this cell means that some 'impurities' must be present in the layer—possibly indicating that the vacuum was not sufficiently good at the time of evaporation. The cell could only be outgassed at 250° c. as higher temperatures would have involved the likelihood of evaporating the materials. However, to produce the correct rate of evaporation the tungsten heater spiral has to be run at about 1,500° c.; this would cause some evolution of gas which might well produce the observed results.

The results obtained with cell B are shown in Figure 3. The short wavelength peak corresponds to the main band of cell A, but there is now also a well defined band of sensitivity with a 'threshold' wavelength $\lambda_{1/2} = 2{\cdot}6\,\mu$.

The limiting sensitivity of this cell at 1·3 $\mu$ was $2 \times 10^{-7}$ watts (i.e. signal = noise in 1 c/s. bandwidth at 80 c/s.)

### 3.6. *Resistance–Temperature Measurements*

The inside of the cell was partly filled with mercury, and this could be heated electrically. Resistances were measured by battery and galvanometer, and temperatures by a thermocouple in the mercury. The results for cell A are plotted as log (resistance) against reciprocal temperature in Figure 2 (*a*). The points fall on a straight line characteristic of semiconductors, which has a slope of 0·25 ev., indicating an activation energy of 0·5 ev. This is presumably related to the 'impurity' band of sensitivity in the 2·6 $\mu$ region. It was not possible to reach the range of intrinsic conductivity as higher temperatures caused permanent changes in the resistance of the layer.

The specific resistance at 21° c. was found to be 110 ohm cm. for cell A, and 32 ohm cm. for cell B, assuming a layer thickness of 1 $\mu$ in each case. The thickness of the layers was estimated from the weight of material used, the area of the layer formed, and the specific gravity, determined experimentally to be 3·4. With the size of cell used, the layer thickness would be nearly 1 $\mu$ per milligram of $Mg_3Sb_2$.

We may calculate roughly what the intrinsic conductivity would be at this temperature. Taking as the density of conducting electrons $n = 2{\cdot}5 \times 10^{19} \exp(-\epsilon/2\boldsymbol{k}T)$ with $\epsilon = 0{\cdot}8$ ev., and assuming a mobility $b = 300$ cm. sec$^{-1}$/volt cm$^{-1}$, which is the order found in recent measurements on ZnO (Hahn 1949) and silicon (Pearson and Bardeen 1949), we find $\sigma = 10^{-4}\ \Omega^{-1}\ \mathrm{cm}^{-1}$. Since the measured conductivity is about 100 times greater than this, it must be mainly due to 'impurity' levels. Hence the activation energy of 0·5 ev. found for this temperature region must be the 'impurity' activation energy, and is thus related to the 2·6 $\mu$ band of sensitivity.

For cell B the measurements were extended down to 160° K. The resulting curve of log (resistance) plotted against temperature (Figure 5) is again linear over a wide temperature range, and indicates an activation energy of 0·48 ev. This is the quantum energy equivalent to a wavelength of just under 2·6 $\mu$—which is remarkably close to the 'threshold' wavelength found from the photoconductivity curve.

It would thus appear that for this material there is little difference between the thermal and optical activation energies, although a difference would be expected in consequence of the Franck–Condon principle. From the theory put forward by

Mott and Gurney (1948), this would imply that the high and low frequency dielectric constants have approximately the same values.

The conclusions to be drawn from the measurements on these two cells are:

(*a*) That there is a band of intrinsic photoconductivity extending from the visible region to a 'threshold' wavelength $\lambda_{1/2}=1{\cdot}53\,\mu$. The intrinsic band is not so well defined for cell B because of the large sensitivity of the $2{\cdot}6\,\mu$ band, but may be isolated by subtracting from the measured total the estimated form of the $2{\cdot}6\,\mu$ band at shorter wavelengths. Assuming that the sensitivity of this band is constant at the plateau value, or of constant quantum efficiency at the plateau value, the resultant curve in either case gives $\lambda_{1/2}\simeq 1{\cdot}5\,\mu$. The values of the intrinsic activation energy are: optically $1{\cdot}53\,\mu$ and $1{\cdot}5\,\mu$ approximately, giving 0·81 ev., and thermally 0·8 ev. approximately, from Boltaks and Zhuze (1948).

(*b*) An 'impurity' band appears in both cells, but is better defined in cell B, where the photoconductivity curve gives $\lambda_{1/2}=2{\cdot}62\,\mu$ (0·47 ev.). Thermal measurements on the two cells give an activation energy of 0·5 or 0·48 ev.

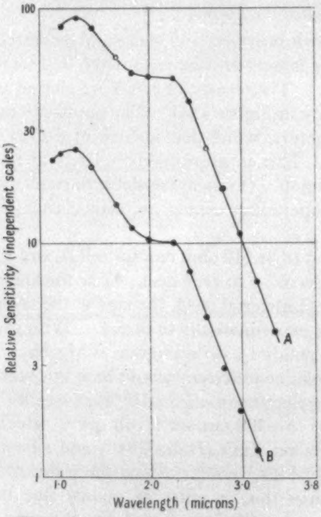

Figure 3. Spectral sensitivity of cell B. Curve A, 290° K. Curve B, 195° K.

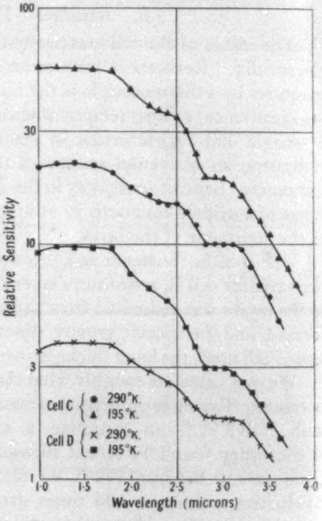

Figure 4. Spectral sensitivity curves.

It is likely that this band corresponds to the band of sensitivity found in Cd$_3$As$_2$ at $2{\cdot}3\,\mu$. The wavelengths to which Zn$_3$As$_2$ and Mg$_3$As$_2$ would be sensitive would be expected to decrease progressively from $2{\cdot}3\,\mu$, since the cation polarizabilities are smaller. The inference is, therefore, that antimonides are photoconductive at longer wavelengths than the corresponding arsenides.

## § 4. SPECTRAL SHIFT WITH TEMPERATURE

The two spectral sensitivity curves shown in Figure 3 were measured on cell B at room temperature and at the temperature of solid CO$_2$ in acetone respectively. Both the threshold wavelengths (half the plateau value) lie at $2{\cdot}62\,\mu$ as read from the curves. A repetition of the experiment gave an apparent shift with temperature of 0·02 $\mu$, which is less than the overall error of the measurements. We may say therefore that no definite shift of the threshold wavelength was observed.

From the theory put forward to explain the spectral shift in PbS, PbTe etc. (Moss 1950) it was shown that a relationship between the spectral shift, threshold energy $E$, and expansion coefficient $\beta$ would be expected, namely,

$$\frac{dE}{dT}E^{-1/2}\beta^{-1}=\text{constant.}$$

For PbS and PbTe the constant was found to be approximately 30. Taking this value to apply to $Mg_3Sb_2$, we would expect the relation $dE/dT=30\beta E^{1/2}$.

The value of $\beta$ was determined experimentally using a direct reading expansion indicator with a sample of $Mg_3Sb_2$ 12 mm. in length. The result obtained was $\beta=9\times10^{-6}$.

Hence $dE/dT=0{\cdot}018$ ev/°c. For the 95° c. change in temperature this would be equivalent to a wavelength shift of 0·09 $\mu$. Such a shift was not observed in the measurements.

Hence if the above theory of spectral shift is to apply in this case, there must be a shift of at least 0·05 $\mu$ in the opposite direction, possibly of the type considered by Radkowsky (1948).

## § 5. CELLS C AND D

Two sensitive layers were prepared by evaporation in a somewhat higher pressure of air (estimated at approximately $10^{-2}$ mm. Hg by glow discharge). These layers had rather a black amorphous appearance.

### 5.1. *Spectral Sensitivity Measurement*

Spectral sensitivity curves were plotted for both cells at room temperature and 195° K. The curves are shown in Figure 4. All curves show the presence of an additional band of sensitivity, with a half-value threshold wavelength near to 3·5 $\mu$ for cell C and 3·6 $\mu$ for cell D. The other two band edges occur at somewhat longer wavelengths than in the previous cells—possibly due to a reduction in activation energies caused by higher impurity content (Heywang 1949). The conditions under which the cells were made make this seem likely.

### 5.2. *Resistance–Temperature Measurement*

Cell D was the only one made which could be cooled to liquid air temperature without causing any permanent change in the layer resistance. On cooling to 77° K. the layer resistance increased to $3\times10^{10}$ ohms. The process was reversible and repeatable. For the electrode dimensions used and a layer 1 $\mu$ thick this value corresponds to a specific resistance of $0{\cdot}9\times10^{7}$ ohm cm. The specific resistances of the constituent metals at 90° K. are: Mg, $1\times10^{-6}$ ohm cm., Sb, $10\times10^{-6}$ ohm cm.

Hence we find that the specific resistance of the compound has increased by the remarkable factor of $10^{12}$ times over that of its constituents. It is to be concluded therefore that a true compound is formed, and that from the point of view of electrical properties the compound more nearly resembles an insulator than a metal.

The resistance temperature curve for cell D is shown in Figure 5. The high temperature part of the curve has an activation energy of 0·46 ev., which is approximately the same value as that found for cells A and B. At lower temperatures there is another linear part of the curve with an activation energy of 0·37 ev. This is not so well defined as the high temperature region, and the slope determination is not so accurate. However, a further energy level is indicated, and this would be expected to correspond to the long wavelength band of sensitivity found in these two latter cells.

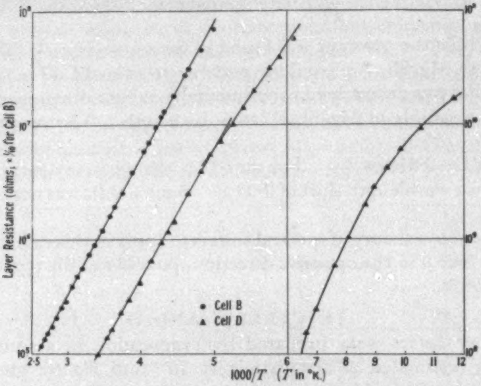

Figure 5. Resistance temperature curves.

The values of activation energy found are: optically 3·5 μ and 3·6 μ, giving an average 0·35 ev., and thermally 0·37 ev., The activation energies of the three bands of sensitivity may thus be taken as 0·81 ev., 0·48 ev. and 0·35 ev.

## §6. POSSIBLE NATURE OF THE IMPURITIES

The term 'impurity' is used in a very wide sense in this discussion, and does not necessarily refer to foreign atoms. Any lattice imperfections might produce energy levels in the forbidden zone and lead to the results observed. The materials used to make the layers were of high purity, and care was taken to prevent contamination during preparation. Hence the only foreign atoms likely to occur in the layers are those from the gas present during the evaporation, i.e. mainly oxygen and nitrogen. However, the gas may not necessarily combine with the layer, but may merely cause the layer to deposit in a less regular form.

The impurity levels may well be due to a stoichiometric excess of one of the constituents, the excess appearing in the lattice interstitially or as a consequence of vacant lattice sites of the other constituent.

## §7. HARDNESS

It is interesting to note that the compound $Mg_3Sb_2$ is much harder than its constituents. The compound would just scratch natural quartz, indicating a hardness figure close to 7, compared with 2 for Mg and 3 for Sb (*Handbook of Chemistry and Physics*).

## §8. CONCLUSIONS

Photosensitive films of $Mg_3Sb_2$ have been prepared by vacuum evaporation. At low light levels the photo-response is proportional to the incident light intensity.

Layers made in high vacuum show one main band of photoconductivity, with a long wavelength 'threshold' at $1{\cdot}5\,\mu$ or 0·8 ev. This corresponds closely to the value of intrinsic activation energy found from Hall effect measurements by other workers.

When the layers are evaporated in a low pressure of air, two other bands of sensitivity appear. These have threshold wavelengths at $2{\cdot}6\,\mu$ (0·47 ev.) and $3{\cdot}5\,\mu$ (0·35 ev.) respectively. Resistance temperature measurements give activation energies of 0·48 ev. and 0·37 ev. The thermal and optical activation energies are thus nearly equal for all the bands.

No appreciable shift of the threshold wavelength was observed on cooling to 195° K.

Since the work was concluded, a brief abstract has appeared stating that the Russian workers Zhuze *et al.* (1948) have observed photoconductivity in $Mg_3Sb_2$.

## ACKNOWLEDGMENT

Acknowledgment is made to the Chief Scientist, Ministry of Supply, for permission to publish this paper.

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# The Rehbinder Effect

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*ABSTRACT*. According to Rehbinder the immersion of wires of certain metals in non-polar paraffin containing a little oleic acid increases the rate of flow of the metal under a given stress and increases the electrical resistivity. The experiments described show that the mechanical effect can be obtained with single crystals of cadmium if the surface is contaminated by a thin oxide layer, which is known to increase the critical shear stress, but not if the surface is clean. The effect is accordingly attributed to the disruption of the hardening surface layer by the active agent and not to penetration into the metal, as assumed by Rehbinder. It has not been found possible to detect any electrical effects with cadmium or lead crystal wires having either clean or contaminated surfaces.

Rehbinder and his collaborators have described in a series of publications (1931, 1937, 1941) striking effects of a surface active liquid, present in small quantities in a non-polar hydrocarbon liquid, on the mechanical and electrical properties of metals and other substances. The contact of such liquids with polycrystalline specimens of lead, tin and copper is said to accelerate plastic flow under a given stress, and to produce an increase in the electrical resistivity of the metal. Particularly striking results are recorded with single crystals of tin and zinc, of diameter 0·2 to 2 mm., extended in a solution of 0·2 % oleic acid in a non-polar hydrocarbon oil, 0·2 % having been found to be an optimum concentration. This liquid will be called 'Rehbinder's reagent' in this paper. The yeild stress in a particular case is recorded as reduced from 265 to 130 gm.wt/mm$^2$, and the stress needed to maintain a constant rate of extension as reduced by the liquid to the extent of some 50 %, over a range of elongations. Marked changes in electrical resistance were also recorded. In one case the resistance of a crystal extended by 180 % in the liquid was four times that of a similar crystal extended in air, the resistance being measured while the stress necessary to produce the extension was still applied. Rehbinder further found that on removal of the stress the resistance of the crystal slowly diminished, tending towards its normal value. He attributed all these effects to the penetration of thin films of the active medium into minute cracks developed during the deformation. The increase of electrical resistance, for instance, was attributed to the insulating action of the films of active material, the recovery on relaxation of load being due to the gradual healing of the cracks, with expulsion of the liquid films.

Recently D. S. Kemsley (1949) published a brief account of experiments on single crystals of tin, which showed no effect of Rehbinder's reagent on the critical shear stress or on the electrical resistance.

Work in this laboratory has shown the great effect which oxide, or similar, films on the surface of a metal single crystal have on its mechanical properties. Roscoe (1936) first drew attention to the marked effect of oxide films in raising the critical shear stress. Recent experiments on which a preliminary note has appeared (Andrade and Randall 1948) have shown that the contact of certain electrolytes with the surface of cadmium single crystals produces effects which have been traced to surface layers. Crystals loaded to flow at a slow rate have shown an immediate increase in rate of flow of roughly three times when immersed in a solution containing cadmium ions, whereas crystals of which the surface has been thoroughly cleaned by thermal evaporation *in vacuo* show no increase in rate on immersion. These experiments made it seem possible that the mechanical effects observed by Rehbinder might be due to a disintegrating action of the surface agent on such films rather than to a deep penetration of the agent into minute cracks (Andrade and Randall 1949). The following experiments were carried out to investigate this point.

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The crystals used were made from clean spectroscopically pure cadmium wire, 1 mm. diameter, by the Andrade and Roscoe method (1937). The disposition of the crystal axes was found by standard x-ray back-reflection methods: in the diagrams $\psi$ indicates the angle between the glide plane and the wire axis and $\lambda$ the angle between the wire axis and glide direction.

The measurements were made with apparatus already described which permitted the wire, slowly extending under a fixed load, to be surrounded by a chosen liquid at any desired moment. Any weakening effect due to the slow rupture or disintegration of the oxide film would be expected to show as an increase in the rate of flow.

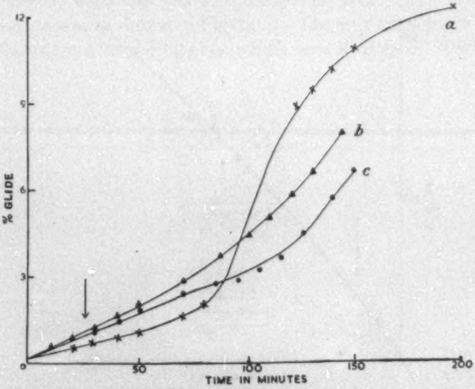

Figure 1. Single crystals of cadmium.

0·2% Oleic acid in paraffin applied at time indicated by arrow: (*a*) oxidized by long exposure to the atmosphere; (*b*) oxidized by heating in oxygen and cleaned; (*c*) oxidized by heating in oxygen and not cleaned.

Resolved shear stress: (*a*) 27·4 gm.wt/mm$^2$, (*b*) 19·1 gm.wt/mm$^2$, (*c*) 24·9 gm.wt/mm$^2$; $\psi=39°$; $\lambda=43°$.

Figure 1 shows the results of the oleic acid mixture on three lengths of crystal, all from the same specimen: (*a*) is a crystal which had been left lying about for some time in air. The surface looked dirty, and no doubt was covered with an oxide film, but this was not visible as a coloration; (*b*) is a crystal which had been covered with a yellow film of oxide by heating in air at about 250° c. for four hours. It was subsequently cleaned by washing with N/10 HCl, water, N/10 NaOH and then water, the method adopted by Rehbinder; (*c*) is a similar crystal which had not been cleaned. The resolved shear stresses on the glide plane and in the glide direction were 25, 18 and 23 gm.wt/mm² respectively.

After about half an hour's immersion, the rate of flow of (*a*) started to increase and eventually reached a value some twelve times the initial rate. About 100 minutes after the introduction of Rehbinder's reagent, hardening began, as normally occurs from the start with a wire of this inclination.

Wire (*b*), rapidly oxidized and cleaned by Rehbinder's method, was the softest. It showed an increase of rate after half an hour, increasing to about 2·5 times the initial rate. Wire (*c*), not cleaned, was a little harder and showed a slightly more marked acceleration effect, the rate increasing to about 3·2 times. It thus appears that the light, but very slowly formed, oxide layer which is built up by exposure to the normal atmosphere produces the greatest mechanical effects.

Figure 2 refers to a crystal whose surface has been cleaned by thermal evaporation, the wire having been heated *in vacuo* by the passage of an electric current until marked deposition of vaporized cadmium was visible on the walls of the containing glass tube (Andrade and Randall 1950). As can be seen, there is no change of rate after the introduction of the liquid except the normal slight diminution—a result in accordance with the supposition that the Rehbinder effect is due to a surface layer of oxide.

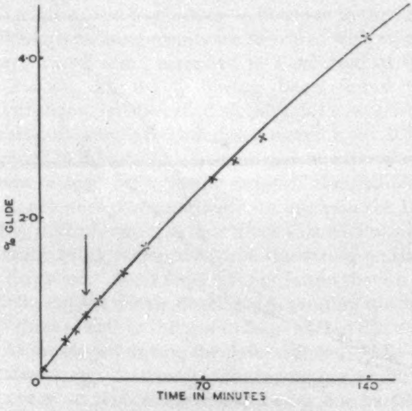

Figure 2. Single crystal of cadmium.
0·2% Oleic acid in paraffin applied at time indicated by arrow. Specimen with surface evaporated *in vacuo*. Resolved shear stress $=30$ gm.wt/mm²; $\psi=21^\circ$; $\lambda=30^\circ$.

Attention was then directed to the electrical effect. In the absence of precise knowledge as to how Rehbinder measured the electrical resistance it seemed possible that the large increase recorded by him might have been caused by the penetration of a film of the active agent between the grips holding the ends of the specimen and the wire itself.

Preliminary experiments were tried with simple apparatus in which the fully extended (100–200 %) crystal was held under tension by springs. The resistance before and after immersion in the Rehbinder's reagent was determined by measuring with a potentiometer the difference of potential between the grips when a steady current was passing through the specimen. No significant increase in resistance could be detected in any experiment, although tests were made on normally prepared crystals of cadmium, oxide-coated crystals of cadmium, and polycrystalline cadmium. Tests were also carried out on oxide-coated lead crystals.

Typical sets of results are given below for oxide-coated cadmium and lead crystals, the times being measured from the introduction of the reagent, so that negative times are before the introduction.

Oxide-coated Cadmium Crystal

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Time (min.) | ... | −40 | −20 | 20 | 40 | 70 | 160 | 260 | 2½ days |
| Resistance* | ... | 2·33 | 2·33 | 2·33 | 2·34 | 2·35 | 2·34 | 2·34 | 2·44 |

Oxide-coated Lead Crystal

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Time (min.) | ... | −22 | −2 | 18 | 38 | 58 | 78 | 93 | 1 day |
| Resistance* | ... | 0·829 | 0·819 | 0·819 | 0·817 | 0·817 | 0·816 | 0·815 | 0·804 |

* in arbitrary units.

In view of the complete failure to reproduce Rehbinder's results it was decided to perform mechanical and electrical measurements simultaneously and the apparatus previously used was modified for this purpose.

The disposition was as shown in Figure 3. The only parts not of metal are the two shaded columns and insulating plug, which were of ebonite. W is the cadmium wire, the load being applied by means of the flexible metal tape H passing over the flat pulley. The connections for the current were the wire E and the flexible tape leading from H to J. Potential leads were soldered to the upper and lower chucks

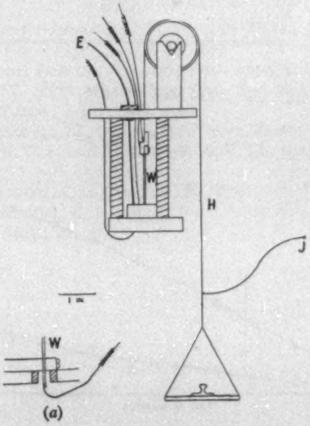

Figure 3.

and also to the ends of the specimens, which passed right through the chucks as shown in inset Figure 3(*a*). Using these leads, changes in potential difference between the ends of the specimen and between the chucks could be measured. The results were expressed as the ratio of these potential differences to the potential difference across a short length of unstrained polycrystalline cadmium wire in series with the specimen: this obviates the necessity for keeping the current constant over long periods of time. In this way changes in resistance of the specimen could be followed and any spurious resistance at the chuck detected. A steady direct current of one ampere was passed through the specimen and the potentials of the various leads measured in rapid succession using a commercial potentiometer reading to 1 microvolt. Successive results were found to differ by less than 5%. No special attempt was made to control the temperature of the apparatus as the magnitude of the effect found by Rehbinder was too large to make this necessary.

While the electrical measurements were in progress the increase of length of the specimen was measured with a travelling microscope.

The results are shown in Figure 4. No Rehbinder effect was observed with a cadmium single crystal prepared *in vacuo* and tested immediately on exposure to the air (curve (*a*)). After oxidation by heating in oxygen for four hours at 200° c. a length of the same crystal gave the curve (*b*) which shows the appreciable hardening due to the oxide and also an increase in the rate of flow rising to 3·3 times the normal rate and commencing some 100 minutes after immersion in the liquid. The slow rise in the electrical resistance of the specimen is entirely accounted for by the change in shape as the extension proceeds. No effect due to the possible penetration of the active agent into the chucks could be detected. Other cases were obtained in which crystals showed a like change in rate of extension without change in resistivity.

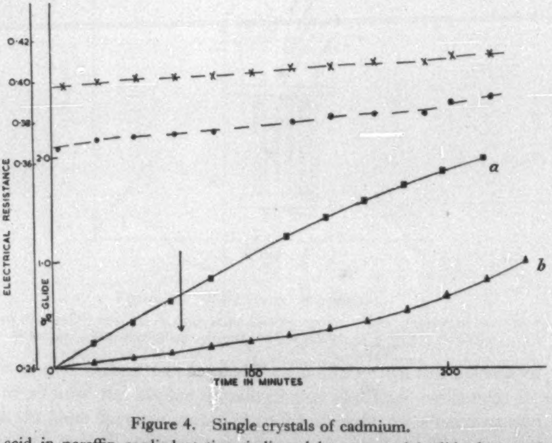

Figure 4. Single crystals of cadmium.

0·2% oleic acid in paraffin applied at time indicated by arrow; (*a*) glide for specimen tested immediately on exposure to the air; (*b*) glide for specimen tested after oxidation.

● Electrical resistance for specimen giving glide curve (*a*); × electrical resistance for specimen giving glide curve (*b*). Resolved shear stress$=33{\cdot}2$ gm.wt/mm$^2$; $\psi=25°$; $\lambda=31°$.

It is clear therefore, that the effects of surface active agents on the mechanical behaviour can occur without any change in resistivity. The fact that we have failed to obtain any marked changes in resistivity suggests that the changes recorded by Rehbinder were associated with some peculiar experimental conditions not essential to the mechanical effect. We conclude that the behaviour of metals in the Rehbinder reagent is due to a disintegration of a surface film and not to deep penetration of the surface active component into minute cracks.

After the work here described was practically completed a paper by S. Harper and A. H. Cottrell appeared (1950) in which experiments were described indicating that the Rehbinder effect is attributable to the action of the reagent on an oxide film. These authors bring strong support to this view by showing a direct proportionality between the viscosity of the reagent and the time taken to produce the effect. They used zinc crystals, with which the effect took place much more rapidly than with our cadmium crystals, in seconds where we found minutes. In view of these completely independent results with two different metals there seems little doubt that the Rehbinder effect as far as mechanical properties of metals is concerned is due to a disintegrating effect on oxide films. Harper and Cottrell made no attempt to detect an electrical effect.

## ACKNOWLEDGMENTS

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# A Study of the Magneto-Resistance of Silicon–Iron

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*ABSTRACT.* Iron containing a small percentage of silicon in solid solution has an unusually small magneto-resistance. Longitudinal and transverse curves of $\Delta\rho/\rho$ against field strength were obtained for a number of single and polycrystalline specimens. In large fields the curves show a linear decrease, which is less than 0·02 times that of nickel and cobalt in the corresponding field range. In small fields various unusual features are found. It is concluded that the small magneto-resistance of polycrystalline silicon–iron is due to the small contribution of individual crystallites and not to an averaging-out process.

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## § 1. INTRODUCTION

THE magneto-resistance, i.e. the change in electrical resistance on the application of a magnetic field, of the ferromagnetic metals and their alloys, can be divided into two distinct parts: a large decrease in resistivity in the neighbourhood of the Curie point owing to an increase in spontaneous magnetization, and a much smaller, more complicated, change associated with technical magnetization curves. The first effect has been studied in detail only in nickel by Potter (1931) and Gerlach (1932) and was interpreted theoretically by Mott (1936). The second effect has been investigated by well over a hundred workers, though single crystal investigations have been reported only by Kaya (1927) and Döring (1938) in nickel, and by Webster (1926, 1927) and Shirakawa (1940) in iron.

Their main results can be described in the following way: the ratio $(\Delta\rho/\rho)_{\parallel}$, i.e. the relative change in electrical resistivity on magnetizing a specimen parallel to the direction of current flow, is increased steadily from zero to a constant final value; on transverse magnetization (perpendicular to the current direction), a steady decrease usually takes place. Both values become practically constant when technical saturation is achieved, and further changes take place only when the applied field increases the spontaneous magnetization appreciably. The technical saturation value of $(\Delta\rho/\rho)_{\parallel}$ for iron is approximately $3\times10^{-3}$ but depends to a considerable extent on temperature, purity and previous heat treatment of the specimen. The $(\Delta\rho/\rho)_{\perp}$ curves for nickel and iron are precisely those one would expect from a large aggregate of randomly orientated single crystals. From the simplicity of the magneto-resistance curves and considerations of cubic symmetry, Döring derived a general relation for $\Delta\rho/\rho$ in terms of five constants $K_1 \ldots . K_5$, and even powers of $\alpha$ and $\beta$, which represent the direction cosines of the magnetization vectors and the current direction respectively, referred to the main crystal axes. It will be shown later that this analysis is not applicable to silicon–iron alloys of certain compositions.

Shirakawa (1936–39) made a study of the longitudinal magneto-resistance of a large number of ferromagnetic alloys as a function of temperature, among them silicon–iron alloys of certain percentages. From his results it is concluded that the saturation value of $(\Delta\rho/\rho)_{\parallel}$ at room temperature changes from positive to negative as the silicon content exceeds 1·7%. Further, it appears that this value is much smaller in the region of 1 to 3% silicon than that of any other ferromagnetic material hitherto measured. This has been confirmed by Drozhima and Shur (1947). Before any explanation of this phenomenon can be offered, it is necessary to investigate whether the small coefficient is due to a genuine reduction in the contribution of individual crystallites. This was one reason for undertaking the present investigation. Owing to the smallness of $\Delta\rho/\rho$ a very delicate measuring technique is required to record accurately changes in resistance of $10^{-7}$ ohm in the specimen under test.

## § 2. EXPERIMENTAL PROCEDURE

The method of measurement employed was governed by the facts that single crystal specimens available were usually in strip or rod form, 2–4 cm. in length, that their magneto-resistance coefficient is smaller by a factor of 10 than that of most other ferromagnetic metals, and that the cross-sectional area of rods could not be reduced below 2 mm$^2$ without introducing appreciable strains on mounting, as well as increasing the magnetic field due to the specimen current to an undesirable value. The actual resistance of specimens was therefore sometimes less than $10^{-2}$ ohm, so that a simple potentiometer or Kelvin double bridge circuit was out of the question. The principle of the method here used has already been described by Bates (1946) in an investigation on high coercivity alloys. It has been found suitable, with only minor modifications, to achieve the extra sensitivity required and is therefore only described very briefly here.

Roughly the principle is as follows: a constant current of approximately 1·018 amp. is sent through the specimen, across which a potential difference of the order of $10^{-2}$ volt is established. The potential difference is balanced, via a subsidiary circuit, against that produced by an accumulator across a suitable potentiometer. A sensitive galvanometer then measures the out-of-balance current when the specimen is placed in a magnetic field. The instrument used for most of the results described was a Tinsley galvanometer, which gave a deflection of nearly 2 mm. at 1·85 metres scale distance, due to a change of $10^{-8}$ volt in the circuit, when used on the most sensitive range. It will be seen, therefore, that magneto-resistance coefficients of $10^{-6}$ are detectable.

The specimen holders used for rods were similar to those described by Bates. For strips a special design was used to give minimum transverse demagnetizing factors and to avoid putting strains on to the specimens when the potential leads were screwed on. It was decided not to solder the latter to avoid possible chemical complications. The circuit was readily calibrated to the desired accuracy.

All silicon–iron specimens were well annealed at 800–850° c. *in vacuo* just before use. It was ascertained for each specimen separately that the process of mounting it did not affect the results significantly owing to unavoidable strains. The magnetic field was supplied by an electromagnet and was measured with a magnetic potentiometer in the longitudinal, and by a search coil in the transverse experiments. Both instruments were previously calibrated in a standard solenoid. The potentiometer measures directly the field acting inside the specimen. In the case of the transverse measurements, a rough correction for the demagnetizing factor could be made if desired, although the uncorrected, applied field is always plotted here. Before each reading was taken, the specimen was carefully demagnetized.

A number of experimental difficulties had to be overcome. To avoid thermo-electric disturbances, the number of junctions of unlike metals was reduced to a minimum and these were well lagged. The specimen had to be shielded from draughts owing to the large temperature coefficient of resistivity of iron. A simple calculation shows that a change of temperature of 0·02° c. would cause a $\Delta\rho/\rho$ of the same magnitude as the magneto-resistance measured in silicon–iron. This was not serious, as a constant temperature had to be maintained only while a reading was being taken, and the time, therefore, could be reduced to approximately 15 seconds when the stray induction pick-up in the galvanometer circuit was counteracted by a compensating coil; but it limited the maximum field that could be applied to specimens to about 10,000 oersteds. Larger magnetizing currents produced objectionable heating effects even in the short time specified. The most serious difficulty was the avoidance of leaks from the magnet (200-volt) supply to the measuring circuit. The resistance between the two had to be maintained at $10^{12}$ ohms and sensitive experiments could not be carried out successfully on days of high humidity.

Before making experiments with the above specimens the sensitivity of the method was tested by preparing a specimen of copper, with a calculated resistance of 0·003 ohm. The change of resistance expected was of the order of $5 \times 10^{-8}$ ohm on applying a field of 10,000 oersteds. This was readily detected above the unavoidable deflections due to leaks, and the results were consistent with those quoted for this field in the Chemical Rubber Company's *Handbook of Physics and Chemistry*.

## § 3. RESULTS AND DISCUSSION

It was verified that, for a polycrystalline and for two single crystal specimens $\Delta\rho/\rho$ did not change (within the limits of experimental error) upon halving or reversing the current through them. Both results are consequences of Ohm's law and have been confirmed for other metals by Webster (1926) and Stierstadt (1930). It was also found that the remanent magneto-resistance was very small, and less than 5% of the saturation change in single crystal specimens. No detectable difference could be observed in the subsequent magneto-resistance, when a sample was brought to the remanent state by a few large or many small current steps. This was put down to the great magnetic softness of silicon–iron when well annealed. The statement was not found to hold for unannealed nickel strips.

Although a smooth $(\Delta\rho/\rho, H)$ curve could always be obtained as $H$ was increased, small spontaneous variations of $\pm 10\%$ were observed in low fields, and of about half this amount in the saturation value, with single crystals. Variations in the case of polycrystals were considerably smaller; they occurred usually, but not entirely, after resoldering or setting the specimen, and could not be improved by successive annealing. They are undoubtedly due to changes in internal strain, which contribute an energy term which easily swamps the small magneto-crystalline energy of silicon–iron. In view of the considerable variations reported in measurements on 'identical' single crystals, it can be concluded that the present measurements show smaller variations than would be expected, if the experiments were repeated with different specimens cut from the same material in the stated directions. The difference between $(\Delta\rho/\rho)_{\parallel}$ and $(\Delta\rho/\rho)_{\perp}$ at saturation is, of course, not affected. A full discussion of these features is given by McKeehan

(1930) and Becker and Döring (1939). It should be noted that the shape of the curves is always the same.

*Polycrystalline Specimen* (1·8% *Si*). Polycrystalline specimens were cut from a sheet of rolled metal which was supplied as 'genuinely polycrystalline'. In view of the possibility of a preferred crystal orientation, two specimens cut perpendicular to each other were examined and it was found that the differences were insignificant. The shapes of the curves were almost exactly the same when the demagnetizing factor was allowed for, a typical curve being recorded in Figure 1. The following features of interest may be noted. There is a small but definite linear decrease in $\rho$ in the region of increase in spontaneous magnetization. It is about 50 times smaller than that for the corresponding field range in nickel (Englert 1932) or cobalt (Bates 1946). It is very likely that this decrease would also occur in pure iron, but no data of sufficient accuracy are available.

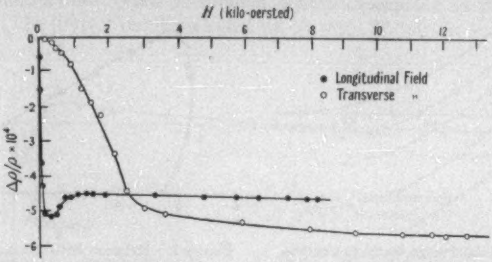

Figure 1. The magneto-resistance of 1·8% silicon–iron, polycrystalline.

The saturation value of $(\Delta\rho/\rho)_{\parallel}$ fits well into the scheme arising from Shirakawa's results for 1·66% and 5·18% silicon–iron at room temperature. The next point of interest is the small but marked minimum in the $(\Delta\rho/\rho)_{\parallel}$ curve in the region of 200–400 oersteds, which could be detected in both specimens measured. No other material well below its Curie point shows such complexity. Its significance will be referred to later on. Another unusual feature is the small value of '$B$' (using Döring's notation), which denotes $(\Delta\rho/\rho)_{\parallel}-(\Delta\rho/\rho)_{\perp}$ at saturation. The only other comparable value is that reported for alnico by Bates (1946). Finally, the relation between longitudinal magnetostriction $\Delta l/l$ and magneto-resistance coefficients is of interest. It was thought for some time that the two phenomena were connected by a simple relation. It has, however, been shown that in a polycrystal a simple relation is demanded by the fact that both phenomena are simple functions of the magnetization vectors with respect to the crystal axes (Becker and Döring 1939). That this relation is of no more fundamental nature is demonstrated by comparing Figure 1 with the magnetostriction curves for the same specimen, Figure 2. The actual value of the magnetostriction coefficient is uncertain to within 30% owing to the unfavourably shaped specimens available, but the shape of the curve, and its variation with tension, is exactly reproducible. The addition of 1·8% silicon has not appreciably changed the magnetostriction properties from those of iron. The magneto-resistance curve, however, is different in shape and magnitude. It is to be expected, therefore, that on plotting $(\Delta\rho/\rho)_{\parallel}$ against $\Delta l/l$ Figure 3, a curve quite different from that for pure iron (Kornetzki 1943), is obtained. It will be noted, however, that in the region of the magnetization curve where rotations from the easy directions do not occur, the two phenomena are proportional to each other, as required from symmetry considerations (Becker and Döring 1939).

*Single Crystal Strip cut along a* [110] *Direction, in a* (001) *Plane* (2·8% *Si*). Actually, the strip plane of this specimen was inclined at approximately 4° to the (001) plane. Results on this crystal are of great importance, for we believe that we now have a reliable picture of the domain processes taking place on magnetization, based on Néel's treatment of the problem. This particular specimen was studied extensively by means of powder photographs by Bates and Neale (1949, 1950) and found to conform to Néel's theoretical predictions. A diagram of the specimen and the magneto-resistance curves obtained are given in Figures 4, 5 and 5 (*a*) respectively.

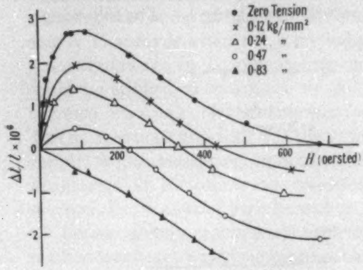

Figure 2. Longitudinal magnetostriction curves of 1·8% silicon–iron, polycrystalline.

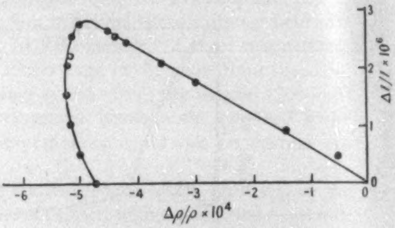

Figure 3. Relation between magneto-resistance and magnetostriction in 1·8% silicon–iron.

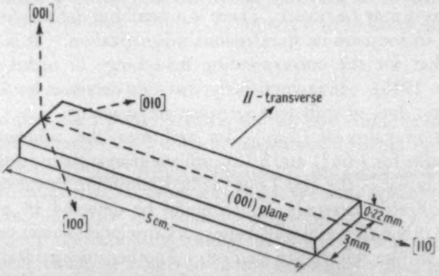

Figure 4. Diagram of single crystal strip of 2·8% silicon–iron.

Apart from the small but definite difference in slope between the longitudinal and transverse curves in very large fields, which is also apparent in Kaya's results on nickel single crystals, the most interesting feature is the complicated behaviour in longitudinal fields below saturation. The initial minimum in the curve is certainly not due to experimental error. It was obtained after repeated annealing, resoldering etc. The possibility of faulty alignment with the field was ruled out by varying the specimen axis by 3° with respect to the applied field, when no appreciable change in the curve resulted. Even if the specimen were turned through a larger angle, the contribution of the transverse effect would here be small, owing to the large demagnetizing factor. It was therefore concluded that the complex curve represents the genuine magneto-resistance contribution. The actual variations in the minimum value were found in a region where considerable variations in the Bitter figure line spacing were observed and must be put down to unavoidable small strains. No curve for other single crystals has shown such complexity, and it is even more surprising when we examine the simplicity of the magnetization process. Apart from 180-degree boundary displacements in the region $H < 20$ oersteds, only domain rotations from the [010] and [100] directions into the [110] direction take place; from considerations of crystal symmetry both types of rotation contribute equally to the magneto-resistance. (The [001] direction is ruled out by shape anisotropy.) The change in resistivity is too complex to be described by Döring's approximation $\Delta\rho = f(\alpha^2, \alpha^4)$, so that for silicon–iron his whole analysis must break down. This is quite apart from the fact that in his equation the constant $\frac{1}{3}$, introduced to allow for the random distribution in the demagnetized state, must be replaced by a function of $\alpha$ and $\beta$, to allow for the shape anisotropy.

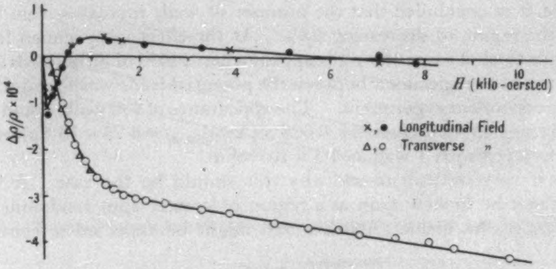

Figure 5. Magneto-resistance curves for single crystal strip of Figure 4, 2 sets of independent measurements.

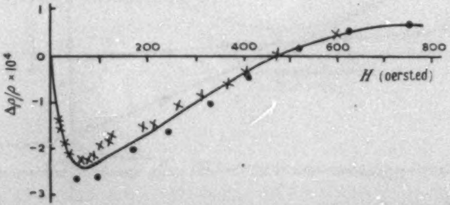

Figure 5 *a*. Part of the longitudinal curve of Figure 5, enlarged.

Two conceivable explanations of this curve are found untenable on close examination. On Néel's theory, certain closure domains on the side of the strip are magnetized parallel to the [001] direction. If changes of the magnetization vectors from [001] to [110] and from [100] to [110] are associated with a decrease and increase in resistivity, respectively, the two opposing effects might produce a minimum. Both from experimental and theoretical considerations we conclude that the volume of such closure domains does not exceed 0·1% of the total volume of the specimen. To make comparable contributions, the magneto-resistance associated with such domains must be more than 500 times that of the other magnetization processes. From Figure 5 it is readily calculated that this would imply a contribution to $(\Delta\rho/\rho)_{\parallel}$ of 0·02, which is several times greater than that for pure iron.

A more reasonable explanation would seem to connect the minimum of the curve with the number of Bloch walls traversed by the current. Bitter figure experiments showed that with the specimen used here, the line spacing at 80 oersteds is approximately 100 $\mu$. Though a line pattern cannot be obtained in the demagnetized state, the number of lines is very much smaller for a very weak field, and the spacing is certainly over 250 $\mu$. This is in reasonable agreement with similar patterns obtained by Williams, Bozorth and Shockley (1949). Since each line indicates the presence of two Bloch walls when the strip surface is not a perfect crystal plane, it is concluded that the number of walls increases from 80 to 200 per cm. in the region of decreasing $\Delta\rho/\rho$. As the effective specimen length was nearly 4 cm., a total of some 480 walls appear when a field of 80 oersteds is applied. The resistance of the specimen between the potential leads was found to be 0·025 ohm by an independent experiment. The appearance of 480 walls would therefore cause a decrease in resistance of $0{\cdot}025 \times (\Delta\rho/\rho)_{80\ \mathrm{oe.}} = 6{\cdot}25 \times 10^{-6}$ ohm, and the decrease associated with 1 wall $\simeq 1{\cdot}3 \times 10^{-8}$ ohm.

Now, is it very difficult to see why this should be the case. A 90-degree Bloch wall must be looked upon as a region of greater spin randomness than a corresponding region inside a domain and might be expected to contribute an increase rather than a decrease in resistance (Mott 1936). Moreover, if we take the length of a unit cell along a [110] direction as 4 A. and the effective thickness of the domain wall as extending over 60 unit cells, we find that the resistivity could be reduced to zero, if a state in the metal could be created so that it consisted entirely of walls.

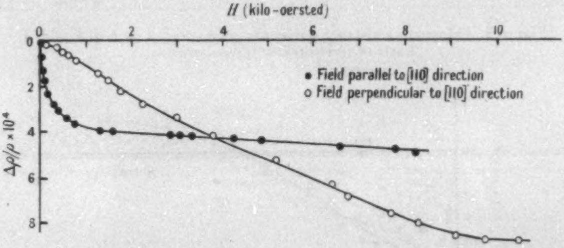

Figure 6. Magneto-resistance of a single crystal strip, specimen surface in (110) plane.

We are led to the conclusion, therefore, that the minimum is due to a rotation of the spontaneous magnetization vectors from a position of 45° to the field into one making a smaller angle. It is reasonable to believe that the slight minimum in the polycrystalline curve is due to the contributions from crystallites of this type. It is hoped, shortly, to discuss elsewhere the reason for this unusually complex behaviour from the point of view of the theory of electrical conductivity in metals.

*Specimen Axis* [110], *Plane* (110), (2·6% *Si*). A specimen was cut along a [110] direction with a rectangular cross section, the longer side of which was contained in a (110) plane. The specimen had an unavoidably large transverse demagnetizing factor, which is demonstrated by the fact that $(\Delta\rho/\rho)_{\perp}$ required an applied field of 10,000 oersteds to reach approximate saturation (Figure 6). The general shape of the curves is not unusual; the actual coefficients are much smaller than for pure iron.

*Specimen Axis* [100], *Plane* (110). (2·6% *Si*). A diagram of the specimen and the curves obtained with it are recorded in Figures 7 and 8 respectively. This specimen also had a large transverse demagnetizing factor, and it is possible that $(\Delta\rho/\rho)_\perp$ at 8,500 oersteds does not represent complete saturation conditions. The horizontal lines in the graph near the origin represent the limits between the values obtained in the region of 600 oersteds in a large number of readings. As

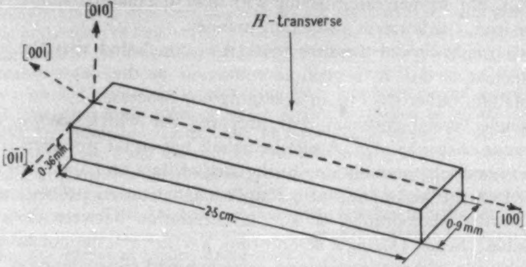

Figure 7. Diagram of single crystal specimen cut along an easy direction of magnetization.

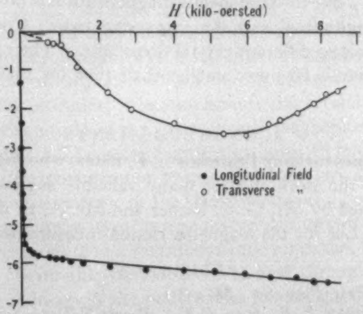

Figure 8. Magneto-resistance curves for specimen of Figure 7.

this specimen had an easy direction of magnetization along its axis we should expect it to saturate in low fields. It will be seen that the change in longitudinal resistance in this specimen reached its final value in a much lower applied field than all the other specimens.

It is interesting to note that the longitudinal coefficient is definitely more negative than the transverse one. Previous workers found, with all materials which they studied, that *under saturation conditions* the quantity $(\Delta\rho/\rho)_{\parallel}-(\Delta\rho/\rho)_{\perp}$ was positive; for the present specimen it is negative under these conditions. The other single crystal specimens described above gave positive values of 2 to $5\times10^{-4}$, whereas the value for the polycrystalline specimen was $1\times10^{-4}$. If we assume that at room temperature there is no significant variation of magneto-resistance with silicon contact from one specimen to another in this work, we must conclude that some crystallites contribute negative saturation values of $(\Delta\rho/\rho)_{\parallel}-(\Delta\rho/\rho)_{\perp}$.

## §4. SUMMARY AND CONCLUSIONS

(*a*) The method described for measuring changes in low resistance can be made sufficiently sensitive to detect changes of $10^{-8}$ ohm.

(*b*) The saturation magneto-resistance of 1·8% silicon–iron fits well into Shirakawa's $(\Delta\rho/\rho)_{\parallel}$ measurements at room temperature for silicon added to iron. The curve has a marked minimum in the region 200–400 oersteds. The value of $B \simeq 10^{-4}$ is unusually small.

(*c*) For 1·8% silicon–iron there is no simple relation between $\Delta l/l$ and $(\Delta\rho/\rho)_{\parallel}$, the former ratio varying with field in a manner similar to that for pure iron, the latter in a different manner.

(*d*) From single crystal measurements it is concluded that the small $\Delta\rho/\rho$ coefficient is due to a genuine reduction in the contribution of each crystallite, rather than to an averaging-out process.

(*e*) A single crystal specimen was prepared for which $(\Delta\rho/\rho)_{\parallel}$ was more negative than $(\Delta\rho/\rho)_{\perp}$. A similar result has so far not been recorded in other materials, possibly due to the lack of data on $(\Delta\rho/\rho)_{\perp}$ for alloys.

(*f*) Based on the assumption of a simple magnetization process, as revealed by powder photographs, no simple correlation between $\Delta\rho/\rho$ and $\alpha$ is possible; hence Döring's assumption, $\rho = f(\alpha^2, \alpha^4)$, cannot be applied to silicon–iron without introducing an unjustified approximation.

(*g*) In agreement with Kaya's results it was found that in a single crystal the decrease in $\rho$ due to spontaneous magnetization is not necessarily the same for the longitudinal and the transverse case. Considerable variations occur also along different crystal directions. The order of magnitude of this reduction is 50 times smaller than that for nickel in corresponding fields.

## ACKNOWLEDGMENTS

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# Investigations on the Reversible Susceptibility of Ferromagnetics

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*ABSTRACT.* A mutual inductometer bridge method has been developed to measure the reversible susceptibility $\kappa_r$ of a ferromagnetic by the application of small alternating fields to specimens in the form of long wires. The corrections which must be applied because of the finite amplitude of the alternating field and for eddy current effects are investigated and an overall accuracy within about 1% is attained. Typical results are given for Swedish iron which show a marked increase in susceptibility after decarburizing and annealing, and this is discussed in relation to the ideas of Néel. By the evaluation of $\int \kappa_r \, dH$ over a suitable range of field, the minimum contribution of reversible processes to the total change in magnetization is estimated and found to vary from 10 to 20% for the specimens of iron and nickel examined. Finally an investigation has been made into the statement of Gans that reversible susceptibility is a unique function of intensity of magnetization and independent of the magnetic history of the specimen; the statement is shown to be true only for specimens which have been brought into a given state of magnetization round a hysteresis cycle, and then only for the regions of higher magnetization.

## §1. INTRODUCTION

THE change with field of the bulk magnetization of a ferromagnetic is due to diverse elementary changes which fall into two main groups. The first group includes continuous movements of domain boundaries through successive equilibrium positions, rotations against the crystal anisotropy forces, and changes in the magnitude of the spontaneous magnetization, all essentially of a reversible character. The second group includes all irreversible boundary movements, whether the movement of an extended boundary across a domain to a position of lower potential energy, or the straightening of a boundary after it has passed some inhomogeneity in its otherwise reversible movement across a domain. These changes are irreversible in a thermodynamic sense too, for they are always accompanied by an irreversible development of heat through the intermediary of the eddy currents induced in the body of the ferromagnetic. The reversible changes approach closely the conditions for strict reversibility in the thermodynamic sense if they are carried out sufficiently slowly, so that eddy current effects are reduced effectively to zero. For irreversible changes, in contrast, once a movement has been initiated it cannot be controlled, and a change occurring during a small increase of field is not reversed by a subsequent corresponding decrease.

Measurements on these two distinctive types of change should yield information which can contribute to a more precise knowledge of the mechanisms by which changes in magnetization occur. Some part, at least, of the irreversible change is associated with the Barkhausen effect, of which a number of experimental studies have been made. Reference to these, and an account of direct measurements of the irreversible changes, are given in papers by Bush and Tebble (1948) and Tebble, Skidmore and Corner (1950); it is shown that, in general, there is a substantial fraction of the whole of the change in magnetization which cannot be ascribed to the Barkhausen processes.

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The present investigations were begun partly to supplement those on discontinuous processes, and partly to furnish information which might be of use in the interpretation of the magneto-caloric data of Bates and others (see Bates 1949 and Stoner and Rhodes 1949). Section 2 of the present paper contains a discussion of some of the previous measurements of reversible susceptibility. In § 3 the methods available for measurement of this quantity are discussed and a full description of one of the methods used in these investigations is given; the effects of eddy currents and demagnetizing factors on the methods are considered in §§ 4 and 5. The results obtained by the use of this method are discussed in § 6 in connection with present ideas on domain structure, with the results of work on the Barkhausen effect and with the implications of Gans's law.

A second method has been developed which is of particular use in investigating variations of susceptibility with temperature. Details of this method and the results obtained in its use, which are of immediate interest in connection with the magneto-caloric effect, will be reported in a separate paper.

## § 2. DEFINITIONS AND OUTLINE OF PREVIOUS WORK

### *Definitions.*

If the magnetization in a specimen changes by an amount $\Delta I$ when the field is altered by $\Delta H$ (this being the field corrected for any demagnetization effects due to the form of the body considered), then a susceptibility may be defined as $\Delta I/\Delta H$. Reversible susceptibility, $\kappa_r$, at a point specified by $(I, H)$ on an initial or hysteresis curve has usually been defined by

$$\kappa_r = \operatorname*{Lt}_{\Delta H \to 0} \Delta I/\Delta H \qquad \ldots\ldots(1)$$

where the field increment $\Delta H$ is in the *opposite* direction to the last increment of the field bringing the specimen to the point $(I, H)$. (The same equation holds for the differential susceptibility, $\kappa_d$, but with $\Delta H$ in the *same* direction.) An accurate measurement of $\kappa_r$ directly on the basis of this definition is obviously, in general, difficult, and in most of the more recent work it has been tacitly assumed that the same reversible susceptibility is measured by the use of alternating fields. The corresponding definition is expressed by the same equation (1), but $\Delta H$ is now the amplitude of an alternating field superposed on the steady field, and $\Delta I$ the corresponding amplitude of the variation in $I$. It is this 'alternating-field' definition which has been adopted in the present work. In so far as $\Delta I$ varies linearly with $\Delta H$ over a not too restricted range of $\Delta H$, the value of $\kappa_r$ obtainable experimentally is unambiguous and accurately measurable. There is no evidence that it differs from the value obtainable in principle from 'direct-field' measurements, but it is perhaps more certainly a reversible susceptibility. It should be noted, however, that non-linearity does not *necessarily* imply irreversibility, and the values obtained, though consistent with a commonly, if tacitly, accepted definition, should strictly be regarded as a lower limit to the reversible susceptibility, in the sense that the integral of $\kappa_r dH$ over any range of $H$ may not give the entire change of $I$ resulting from reversible changes.

The method chosen for the determination of $\kappa_r$ must clearly be such that accurate measurements may be made of the changes of $I$ due to 'very small' changes of field. Precisely how small the change must be to satisfy the reversibility condition is a matter for experimental investigation in the light of the formal relations (e.g. range of linearity of the variations) on the one hand and, on the other, of general knowledge of the character of the elementary processes occurring.

Usually the reversible susceptibility is considerably less than either the 'technical susceptibility' $I/H$ or the differential susceptibility $dI/dH$. At the beginning of the initial magnetization curve, and at high values of $H$, however, the reversible and differential susceptibilities are very nearly equal, reversible changes in these regions being dominant.

*Previous work.*

Ewing (1900) recognized the existence of these different susceptibilities, but the first comprehensive measurements of reversible susceptibility were made by Gans (1911). The results were summarized in the statement (sometimes referred to as Gans's law) that "the reversible susceptibility is a unique function of the magnetization and is independent of the magnetic history of a specimen". Further, a relationship, involving the Langevin function, between the reversible susceptibility at a given magnetization and that of the substance in the demagnetized state was given in terms of a parameter $x$ in the form

$$\left.\begin{aligned} \kappa_r/\kappa_i &= 3\,L'(x), \\ I/I_s &= L(x), \\ L(x) &= \coth(x) - (1/x), \end{aligned}\right\} \qquad \ldots\ldots(2)$$

where $\kappa_r$ and $I$ are the values of reversible susceptibility and magnetization under the conditions of measurement, $\kappa_i$ is the initial reversible susceptibility, and $I_s$ the saturation magnetization.

The equations (2), which give a monotonic decrease of $\kappa_r/\kappa_i$ from unity at $I=0$ to zero at $I=I_s$, were not derived theoretically, but the variation indicated was in reasonably good agreement with a considerable number of Gans's results. A tentative theoretical derivation of the equations was later given by Debye (1925), but the basic assumptions which it appeared necessary to make would not now be regarded as physically valid. More recently, some theoretical support for the relation as an approximation has been provided by Brown (1938, 1939) on the basis of a consideration of reversible boundary movements.

Gans's experimental method was to apply small changes of magnetic field to a specimen and to measure the corresponding change in induction ballistically. The specimens, soft and hard steel and iron and hard drawn nickel, were in the form of ellipsoids. Deviations from Gans's law were ascribed to inhomogeneities in the materials. No tests seem to have been made to ensure that the field changes were sufficiently small to give the true reversible susceptibility. Measurements were made along the initial magnetization curve and round the hysteresis loop, but the region inside the loop was not fully investigated.

The work of Erhardt (1917) is of interest in that he was the first to use an inductance comparison method for reversible susceptibility measurements. Some of his results are in fair agreement with Gans's empirical relation, but he was more concerned with the variation of susceptibility with frequency. No change could be detected up to a frequency of 1 Mc/s. This is a rather surprising result, since eddy current and other effects are known to be appreciable for specimens of the dimensions given.

The work of Sizoo (1929) showed that some of the effects ascribed by Gans to inhomogeneities were due to the use of too large measuring fields. For a ring specimen of laminated silicon iron the Gans relation held much more closely for a field change $\Delta H$ of the order of $10^{-3}$ oersted than for $\Delta H$ of the order of $10^{-1}$ oersted.

Samuel (1928) found that there was a unique functional relationship between reversible susceptibility and magnetization for cobalt, but that it differed markedly from the Gans relation. Brown (1938) has shown that this could be accounted for by anisotropy in the domain orientation. In Samuel's measurements the reversible susceptibility was determined at points along the initial magnetization curve and round the hysteresis cycle taken between near saturation values of magnetization, but not at points inside the hysteresis loop.

None of these previous measurements gave results which could be assumed to hold for the specimens used in the Barkhausen effect measurements of Bush and Tebble (1948) and Tebble, Skidmore and Corner (1950). It is essential, if a complete self-consistent description of the magnetic properties of any substance is to be obtained, that so far as is possible the identical specimens and conditions should be used for the different types of measurement necessary, since slight differences in degree of purity and conditions of strain can give large differences in the structure-sensitive magnetic properties.

## § 3. EXPERIMENTAL METHOD

Any measurement of susceptibility consists essentially of a measurement of the change in magnetization brought about by a change of field. There are two broad groups of possible methods for the measurement of reversible susceptibility. The first of these includes the 'direct' methods in which the change of magnetization is measured with a ballistic galvanometer or a magnetometer. These methods are by no means ideal for the present purpose since it is necessary to use instruments of a high sensitivity in order to detect the small changes in intensity produced by very small changes of field (a thousandth of the coercive force or less).

The second group includes methods based on measurements of the inductance of a coil with and without the specimen under test as a core. The inductance of the coil is obviously a function of the susceptibility of the material of the core, and this can be calculated from the measurements. A variation of this method, which has considerable practical advantages, is the measurement of the mutual inductance of two coils carrying the specimen. The mutual inductance may be balanced against a standard in some form of A.C. bridge circuit and so directly determined, or the voltage induced across the secondary coil may be measured. The bridge system has been used in the work described in this paper. This group of methods has the advantage that within limits the sensitivity increases with the frequency of the applied voltage; eddy current effects, however, set an upper limit to the frequency which can advantageously be used.

The full bridge circuit diagram is shown in Figure 1 and a simplified circuit in Figure 2.

The circuit, based on a Campbell mutual inductometer, is similar to one used by Hartshorn (1925) for mutual inductance measurements and has the advantage of giving direct readings.

In Figure 2, $M_c$ and $M$ are the mutual inductances of the inductometer and coil assembly respectively. The conditions of balance are

$$M = M_c \qquad \ldots\ldots (3a)$$

$$r + \sigma_1 + \sigma_2 = 0, \qquad \ldots\ldots (3b)$$

where $\sigma_1$ and $\sigma_2$ are effective resistances giving rise to induced voltages in the secondary in quadrature with the purely inductive E.M.F.'s ; the effect of $\sigma_1$ and $\sigma_2$ is balanced by the variable resistance $r$. The value of the mutual inductance $M$ can be obtained directly from a reading on the inductometer.

In Figure 1 the balance is attained by adjusting the resistance circuit $R_3$ and $R_{4a}$, $R_{4b}$ until ($3b$) is satisfied, and then the inductometer until ($3a$) is satisfied. As $\sigma_1$ and $\sigma_2$ may be such as to give rise to a voltage leading or lagging with respect to the inductive component, the switch S enables the balancing potential to be reversed. The current passing in the primary circuit, and hence the magnitude of the applied alternating field, can be measured. Using an A.F. amplifier and headphones, the inductometer can be balanced with an accuracy of better than 1 in 1,000.

The coil assembly consists of a ' secondary ' coil wound on a glass tube inside which the specimen is placed. This is mounted coaxially inside a long solenoid (the 'primary' coil) which gives a uniform field over the volume enclosed by the secondary coil. A third coil (the ' polarizing ' coil) wound over the primary coil enables a steady field to be applied to the specimen so as to bring it to any desired point on the magnetization curve; this coil is also used to demagnetize the specimen by means of a slowly reduced alternating current. The use of a choke of high impedance in the supply circuit to the polarizing coil renders negligible the effect of any impedance from this source reflected into the mutual inductance circuit. The assembly is carefully positioned so that the axis of the specimen is at right angles to the earth's magnetic field. The secondary coil is removable and various coils of different diameters and numbers of turns are used to give the required sensitivity for different specimens.

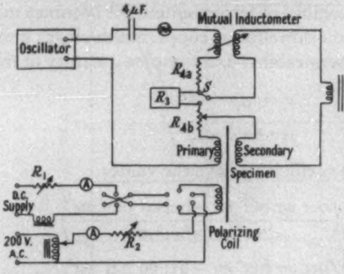

Figure 1. Bridge circuit for measurement of reversible susceptibility.

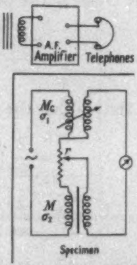

Figure 2. Simplified bridge circuit.

It may be simply shown that the reversible susceptibility with a coil assembly such as the above is given by

$$\kappa_r = \frac{A}{4\pi a}\left(\frac{M_2}{M_1} - 1\right) \qquad \ldots\ldots(4)$$

where $M_2$ and $M_1$ are the values of mutual inductance with and without a specimen present respectively, $A$ is the mean area of cross section of the secondary coil and $a$ that of the specimen.

The accuracy obtainable in the measurement of $M_2/M_1$ with this apparatus, at least 1 in 1,000 (Rayner and Ford 1950), is much better than can normally be obtained by the alternative method of measuring the voltage induced in the secondary coil. However, the former method is too slow to allow of its use in the susceptibility–temperature measurements, and it may be appropriate to mention here that a rapid automatic method has been devised for this purpose. The voltage induced in the secondary coil is amplified, rectified and displayed as the $y$ deflection of a cathode-ray oscillograph, while the voltage across the polarizing coil provides the $x$ deflection as the specimen is taken through a hysteresis cycle; a $(\kappa_r, H)$ curve is thus traced on the cathode-ray tube screen and recorded photographically. Fuller details will be given in the later paper.

## § 4. EDDY CURRENT EFFECTS

The assumption has so far been made that the variations in $B$ and $H$ are exactly in phase. This is not generally the case since eddy currents retard the flux change and make the distribution of flux across the specimen non-uniform. The effect is analogous to the skin effect in conductors carrying high-frequency currents, but owing to the high permeability of ferromagnetics it becomes marked at much lower frequencies than does the skin effect in copper conductors; it may be formally represented by treating the permeability as a complex quantity of the form

$$\mu = \mu_0(\alpha + i\beta)$$

with

$$\mu = 4\pi\kappa + 1.$$

It may be shown that the coefficients have the values

$$\alpha = \frac{2}{mr}\left(\frac{\operatorname{ber} mr \operatorname{bei}' mr - \operatorname{bei} mr \operatorname{ber}' mr}{\operatorname{ber}^2 mr + \operatorname{bei}^2 mr}\right),$$

$$\beta = \frac{2}{mr}\left(\frac{\operatorname{ber} mr \operatorname{ber}' mr - \operatorname{bei} mr \operatorname{bei}' mr}{\operatorname{ber}^2 mr + \operatorname{bei}^2 mr}\right),$$

where $m = (4\pi\omega\mu_0/\rho)^{1/2}$, $r$ is the radius of the specimen, $\rho$ the resistivity, and $\omega/2\pi$ the frequency of the alternating field (Scott 1930).

The voltage induced in the secondary coil is thus a complex quantity, but only the reactive portion of this voltage is balanced by the mutual inductometer and so measured. Thus the true reversible susceptibility is given by the formula

$$\mu_0 = \frac{1}{\alpha}\left[\left(\frac{M_2}{M_1} - 1\right)\frac{A}{a} + 1\right].$$

In a method which measures the actual voltage ($V_2$ and $V_1$) induced across the secondary it is necessary to take into account both components, and this leads to the expression

$$\frac{V_2 - V_1}{V_1}\frac{A}{a} = [\mu_0{}^2\beta^2 + (\mu_0\alpha - 1)^2]^{1/2}.$$

A close approximation for all actual cases is given by

$$\mu_0 = \frac{1}{(\alpha^2 + \beta^2)^{1/2}}\left(\frac{V_2 - V_1}{V_1}\frac{A}{a} + 1\right). \qquad \ldots\ldots(6)$$

The functions $\alpha$ and $(\alpha^2 + \beta^2)^{1/2}$ are thus a measure of the eddy current correction factors in each case and are shown in Figure 3, plotted against the parameter $mr$.

The curves are 'universal' and may be applied to any cylindrical specimen for which the demagnetizing factor is small.

## § 5. EFFECTS DUE TO DEMAGNETIZING FIELD

The effective susceptibility of the specimen is reduced by the effect of the demagnetizing field. The relation between the true reversible susceptibility, $\kappa_r$, and the effective value is given by

$$\kappa_r = \kappa_e\left[\frac{1}{1-N\mu_e/4\pi}\right],$$

or

$$\kappa_r \simeq \kappa_e\left[\frac{1}{1-N\kappa_e}\right],$$

where $N$ is the demagnetizing factor.

To meet the conditions for small eddy current corrections (i.e. small values of *mr*) the specimen must usually have such a small demagnetizing factor that the effect on the reversible susceptibility measurement is negligible. In the original design of the apparatus this was considered, and it was constructed to take wire specimens of great (length/diameter) ratio, so that the demagnetizing factor is very small.

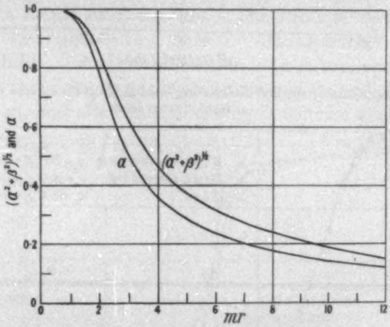

Figure 3. Eddy current correction curves.

## § 6. EXPERIMENTAL RESULTS

(*a*) Great care was always taken to ensure that the alternating field was small enough for the quantity measured to be a truly reversible susceptibility. The points at which there is the greatest danger of this field being too high are at the foot of the initial magnetization curve and at values of the polarizing field of the order of the coercive force, i.e. where values of $\kappa_r$ are greatest. Under near saturation conditions it is possible to use large alternating fields with little change in the measured susceptibility. This would be expected if reversible processes predominate in that region. If, therefore, the magnitude of the alternating fields is sufficiently low to satisfy the conditions of reversibility at the critical points mentioned it will be satisfactory for all other parts of the hysteresis loop.

Measurements of the reversible susceptibility were made for various amplitudes of the alternating field at the two critical points and a graph plotted in each case. A typical curve obtained for a demagnetized specimen of hard drawn Swedish iron is shown in Figure 4. In this case an accuracy to within 1% would be obtained if an alternating field of less than $10^{-2}$ oersted r.m.s. were employed. In some cases the fields required were lower than could be conveniently used if an accurate balance were to be obtained on the inductometer. In these cases a reading was taken at each of two small values of alternating field amplitude and the value for an infinitely small field obtained by extrapolation, assuming the relation between reversible susceptibility and field amplitude to be linear. This is in fact so over a wide range, in agreement with the Rayleigh law. In the measurements an overall accuracy of within ±1% was aimed at.

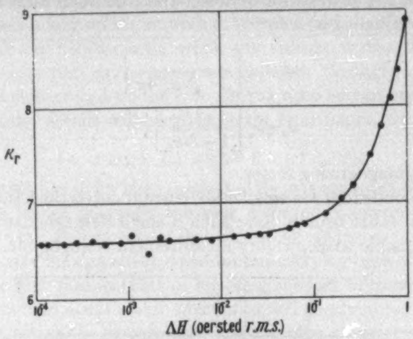

Figure 4. Effect of amplitude of applied alternating field $\Delta H$ on measured reversible susceptibility of hard drawn iron.

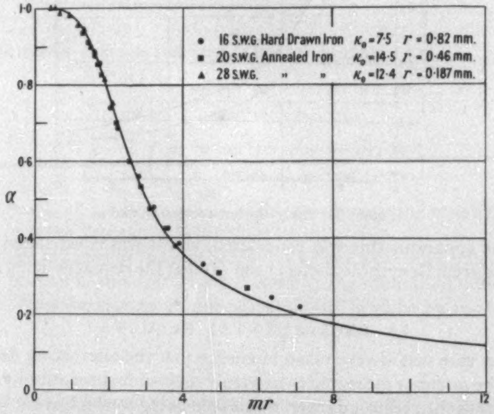

Figure 5. Comparison of calculated and measured eddy current correction factor $\alpha$.

(*b*) It was considered desirable to check the eddy current theory experimentally before using it to correct the measured susceptibility for effects due to the frequency of the alternating field. The susceptibility was measured at frequencies up to about 10 kc/s. for three specimens of very different dimensions and susceptibility; the results are shown in Figure 5.

The experimental points lie very close to the theoretical curve for values of $mr < 4$. In the measurements reported here $mr$ has not exceeded 3·5, so that application of the theory is entirely justified.

There is no evidence that the magnetic after-effect discussed by Becker (1939) and Snoek (1947) plays any significant part in the variation of susceptibility under the conditions employed for these measurements. If such an effect were operative it would be expected to cause a further reduction of susceptibility as frequency increased, whereas in fact the susceptibility is rather higher in that region than would be expected on theoretical grounds.

(*c*) Measurements have been made on a number of materials, but a general indication of the nature of the results is shown in those given for Swedish iron in different stages of annealing. The original specimen, in the form of 26 s.w.g. hard drawn wire, was of composition 99·92% Fe, 0·03% C, 0·05% Mn. It was first decarburized by heating in an electric furnace at a temperature of 950° c. for 48 hours with an atmosphere of moist hydrogen. After cooling it was stretched by 3·25% of its length and then carefully annealed by heating at 850° c. for 72 hours in dry hydrogen so as to induce crystal growth; this has the effect of increasing the grain size by a factor of about 100. The effect of this treatment on the magnetization curves is shown in Figure 6. Measurements of the variation of reversible susceptibility and of magnetization with polarizing field $H$ were made on the specimen at three stages in the treatment: in the initial hard drawn state, after decarburization, and after final annealing to increase grain size.

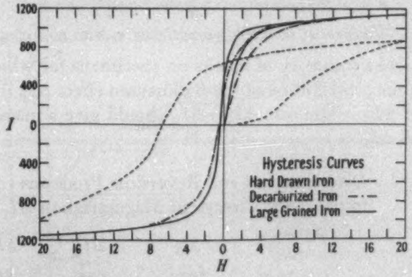

Figure 6. Hysteresis curves for hard drawn iron, decarburized iron and large grained iron.

The results, shown in Figure 7, give the characteristic $(\kappa_r, H)$ curves. The exact form of the $(\kappa_r, H)$ relation has been discussed by other workers (see § 2), but a detailed consideration of the problem is deferred until the susceptibility–temperature results are presented.

The effect of the decarburizing treatment, which must have included some degree of annealing, is very noticeable and is considerably greater than the effect of the subsequent increase in grain size. This is in accordance with the general ideas, which, as developed quantitatively by Néel (1947), lead to the conclusion that in iron the coercive force, and hence the reversible susceptibility, is affected more by the presence of inclusions of foreign material than by internal strains.

The results for nickel are of a similar character but with values of susceptibility in general somewhat lower; in a well annealed specimen for instance $\kappa_r \sim 10$, with correspondingly lower values for cold-worked specimens. In all the materials examined the reversible susceptibility was found to be extremely structure-sensitive, and great care was necessary in handling specimens, particularly those which had been subjected to any degree of annealing. In fact the measurement of reversible susceptibility in the region $H \sim H_c$ is a very delicate test as to whether any significant structural change has taken place in a material.

(*d*) A feature of interest in susceptibility measurements lies in the fact that the area under a $(\kappa_r, H)$ curve, given by

$$\int \kappa_r \, dH = \int \frac{dI_R}{dH} \, dH = \Delta I_R$$

with appropriate limits, is the contribution of the reversible processes to the total change in magnetization over the range of field considered.

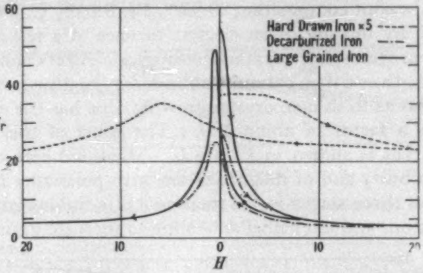

Figure 7. Variation of reversible susceptibility $\kappa_r$ with polarizing field *H*.

The Table gives a summary of results on specimens for which information is also available on the contribution of the Barkhausen effect $\Delta I_B$ to the total change in magnetization $\Delta I_T$. The sum $\Delta I_R + \Delta I_B$ should give a magnetization change equal to the total $\Delta I_T$.

Contribution by Discontinuous and Reversible Processes ($\Delta I_B$ and $\Delta I_R$) to the Total Change in Magnetization $\Delta I_T$.

| Material | Limits of field | $\Delta I_T$ | $\Delta I_B$ | $\Delta I_R$ | $\Delta I_B + \Delta I_R$ |
|---|---|---|---|---|---|
| Hard drawn iron | ±18·6 | 2150 | 1844 | 180 | 2024 |
| Large grained iron | ± 3·0 | 2180 | 1250 | 110 | 1360 |
| Annealed nickel | ±14·9 | 700 | 326 | 115 | 441 |

It will be seen that only in one case is the sum of $\Delta I_B$ and $\Delta I_R$ closely equal to $\Delta I_T$. This failure to account for the whole of the change in terms of the measured reversible and irreversible changes has been discussed by Tebble, Skidmore and Corner (1950). The main reason is considered to be the insufficient sensitivity of the apparatus used for the detection of the Barkhausen effect in the large grained iron and annealed nickel specimens; the minimum detectable volumes correspond to $0{\cdot}87 \times 10^{-10}$ cm$^3$ and $3{\cdot}1 \times 10^{-10}$ cm$^3$ respectively. It is thought probable that the effect of the annealing and increase in grain size in the preparation of the large grained iron is to reduce the number of the larger Barkhausen discontinuities and to increase the number of the smaller irreversible processes which lie below the range of the apparatus. There is no indication that the method of estimating the reversible contribution is seriously at fault, though, as mentioned earlier (§2), the total reversible change in $I$ may be somewhat greater than that estimated from the measured reversible susceptibility.

(*e*) A series of measurements designed to test the validity of the Gans relation was carried out over a wide range of conditions, as shown in Figure 8. This represents the upper half of a series of hysteresis cycles round which a specimen of 26 S.W.G. hard drawn iron was taken, the reversible susceptibilities being measured at suitable points on each curve. Besides the normal type of hysteresis loop, with field limits of 100, 50, 25 and 10 oersteds, measurements were also carried out on the two subsidiary loops shown in Figure 8; a further path on the $(I, H)$ diagram was also traversed from $H = -5{\cdot}2$, $I = 400$ to $H = +14{\cdot}1$, $I = 840$, as shown in Figure 9. Using these results, a contour diagram (Figure 9) was constructed, the contours joining points of equal reversible susceptibility. If Gans's law were strictly true all the contours would be straight lines parallel to the $H$-axis. This is approximately true for the results of the measurements on the loops for higher values of magnetization, but for the lower values the divergence from linearity is marked.

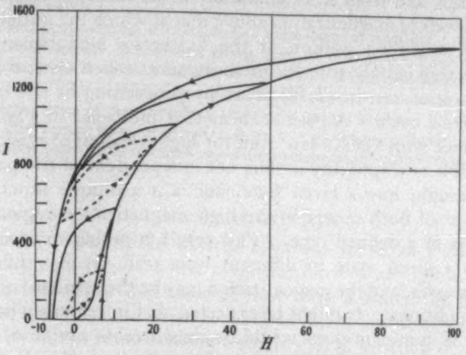

Figure 8. Magnetization curves for hard drawn iron.

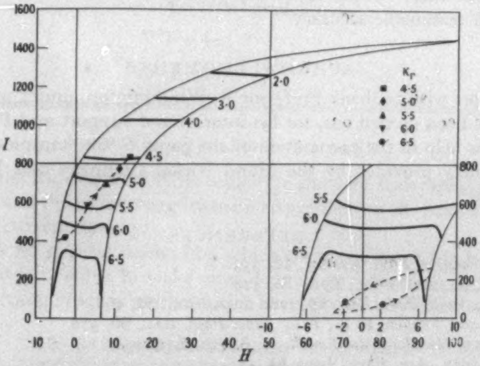

Figure 9. Contour diagrams for hard drawn iron.
Values of susceptibility are shown by figures against contours.

The results of the measurements on the path $(-5{\cdot}2, +400)$ to $(+14{\cdot}1, +840)$ and those on the smaller of the subsidiary loops are shown separately in Figure 9, as the disagreement in these cases is particularly noticeable. On these curves the points at which the susceptibility is of the same magnitude as on a contour are shown using a characteristic mark.

Now the importance of Gans's law lies not so much in the form of the equation (2) *per se* but in the statement that $\kappa_r$ is a unique function of $I$. It would appear from these results, and from an examination of the susceptibility curve, that this holds only for levels of magnetization above that at which the initial magnetization curve and the ascending portion of the hysteresis loop become concurrent. However, even this is only true for measurements taken on specimens brought into a given position on the $(I, H)$ plane by proceeding in the normal manner round a hysteresis cycle. A path such as that indicated in Figure 9 will give results at variance with Gans's law even for higher values of magnetization.

The reversible susceptibility is thus not independent of the path by which a specimen is brought into a given state, nor is it a unique function of field or magnetization or of both except in the high magnetization region or where the path followed is of a defined type. This result is perhaps not surprising since approaches to a given state in different ways will involve different types of elementary processes, and the magnetization may be the resultant of quite different microscopic conditions. It might be expected that in regions where a particular type of change is dominant there would be considerable similarity in the internal state of a specimen to produce a given magnetization; this is found for high magnetizations where reversible effects predominate and where Gans's law is followed with reasonable accuracy.

## ACKNOWLEDGMENTS

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# Congruent Space Charge Flow

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*ABSTRACT.* The paper is mainly concerned with one type of congruent flow, namely irrotational flow, and propositions are established regarding (*a*) rectilinear motion, (*b*) motion in which the current density is constant along lines of flow and (*c*) the representation of lines of flow by the level lines of a harmonic function. From the latter, three corollaries are deduced regarding flow in which the space charge density is constant either throughout the motion, or along lines of flow or along lines of constant action.

Two cases of curvilinear flow, originating from a unipotential cathode, are discussed and are shown to possess important features regarding magnification and transit time.

## § 1. INTRODUCTION

In a large class of electronic devices electrons move as a continuous fluid in a stationary field in which the magnetic force is negligible. The flow is congruent in the sense that lines of flow form a system of curves filling a portion of space such that in general a single curve passes through any given point, the velocity in the motion being a single valued function of position. The equations governing the motion are

$$\dot{\mathbf{v}}=(e/m)\,\mathrm{grad}\,V \qquad \ldots\ldots(1)$$

$$\nabla^2 V=4\pi\rho \qquad \ldots\ldots(2)$$

$$\mathrm{div}\,\mathbf{i}=0 \qquad \ldots\ldots(3)$$

and, since the flow is congruent

$$\dot{\mathbf{v}}=(\mathbf{v}.\nabla)\mathbf{v}=\tfrac{1}{2}\,\mathrm{grad}\,(\mathbf{v}^2)-\mathbf{v}\times\mathrm{curl}\,\mathbf{v} \qquad \ldots\ldots(4)$$

where $V$ is the electric potential, $v$ the velocity, $e$ and $\rho$ the scalar magnitudes of the electron charge and the space charge density respectively, $m$ the mass of an electron and $i(=\rho v)$ the current density.

There exists no general method by which these equations can be integrated rigorously but it is possible to make some deductions concerning particular solutions and concerning types of motion which have specific physical characteristics.

## § 2. CLASSIFICATION OF THE FLOW

There are two characteristic features of the flow, namely that the circulation of the velocity round any closed circuit moving with the flow is constant (as in hydrodynamics, see for example, Weatherburn 1937) and that along a line of flow the sum of the potential and kinetic energy of an electron is constant (see, for example, Meltzer 1949). In a recent paper Meltzer (1949) has distinguished two fundamental types of congruent flow which he has named normal and abnormal, the former being defined as a flow in which the sum of the potential and kinetic energy is everywhere constant. He has shown, moreover, that a necessary and sufficient condition for 'normal' flow is that $\mathbf{v}\times\mathrm{curl}\,\mathbf{v}$ is everywhere zero.

There are other ways in which the flow can be classified. For example, in geometrical terms a normal, as distinct from a skew, congruence refers to the case in which there exists a singly infinite family of surfaces which are cut orthogonally by the lines of flow. A necessary and sufficient condition for this is that $\mathbf{v}.\mathrm{curl}\,\mathbf{v}$ vanishes, as may easily be deduced from geometrical considerations (Synge 1937). Alternatively, in dynamical terms one may distinguish between irrotational and rotational (or vortex) motion, the condition for the former being curl $\mathbf{v}=0$. Clearly, these three ways of classification differ only in their terms of reference and there is no question of one being more fundamental than the others. In fact they are practically identical, the last being the most stringent since it is contained in the other two.

The limitations on the practical realization of congruent flow are very considerable since in all known processes of emission electrons have a statistical velocity distribution rendering such flow impossible unless it can be assumed that the initial velocity is negligible. It is sometimes assumed that electrons are ejected normal to the cathode with a constant velocity (depending on the cathode temperature). This may partially be justified in dealing with rectilinear motion but has no meaning in any other case.

Only two types of zero velocity emitter can be envisaged, an equipotential surface such as a normal indirectly heated thermionic cathode, or a varying potential surface such as a varying potential directly heated cathode. For the former the flow is ‘normal’ in Meltzer’s sense since at the emitter the sum of the potential and kinetic energy is constant. Furthermore, if curl $\mathbf{v}$ exists it must be directed along a line of flow and hence vortex tubes coincide with tubes of flow. The vorticity is measured by the circulation of the velocity round any closed circuit lying on a vortex tube and circumscribing it once. Such a circuit can be found which also lies on the emitting surface and consequently the vorticity is zero. All three normality criteria are thus satisfied. Likewise if the emitter is not an equipotential, curl $\mathbf{v}$ has a value (becoming infinite at the emitter) and the flow is ‘abnormal’, skew and vortex.

### § 3. IRROTATIONAL FLOW

In irrotational flow the velocity can always be expressed as the gradient of a scalar point function, namely the action function $W$,

i.e.
$$\mathbf{v}=\mathrm{grad}\,W. \qquad \ldots\ldots(5)$$

By combining (1), (4) and (5) it follows that

$$\mathrm{grad}\,(\mathrm{grad}\,W)^2=(2e/m)\,\mathrm{grad}\,V$$

hence
$$(\mathrm{grad}\,W)^2=(2e/m)\,V+C \qquad \ldots\ldots(6)$$

where $C$ is a constant which may be put equal to zero without loss of generality, the consequence merely being that speeds are zero when the potential is zero.

In the case of two-dimensional motion a necessary and sufficient condition that (3) be satisfied is that a stream function $\psi$ exists such that

$$\left.\begin{aligned} i_x&=\rho\frac{\partial W}{\partial x}=\frac{\partial\psi}{\partial y}\\ i_y&=\rho\frac{\partial W}{\partial y}=-\frac{\partial\psi}{\partial x}\end{aligned}\right\} \qquad \ldots\ldots(7)$$

where $i_x$ and $i_y$ are the rectangular components of the vector current density. Each line $\psi=$const. is orthogonal to any line $W=$const. and so the functions $\psi$

and $W$ may be used as a reference system of orthogonal curvilinear coordinates. By referring to the elementary cell shown in Figure 1

$$dS_1 = h_1\, dW;\ \ dS_2 = h_2\, d\psi \qquad \ldots\ldots(8)$$

where (Stratton 1940)

$$h_1{}^2 = \left(\frac{\partial x}{\partial W}\right)^2 + \left(\frac{\partial y}{\partial W}\right)^2;\quad h_2{}^2 = \left(\frac{\partial x}{\partial \psi}\right)^2 + \left(\frac{\partial y}{\partial \psi}\right)^2.$$

From equations (7) it follows that

$$i^2 = \rho^2\left[\left(\frac{\partial W}{\partial x}\right)^2 + \left(\frac{\partial W}{\partial y}\right)^2\right] = \left(\frac{\partial \psi}{\partial x}\right)^2 + \left(\frac{\partial \psi}{\partial y}\right)^2 \qquad \ldots\ldots(9)$$

and hence, by expressing $\partial x/\partial W$, $\partial y/\partial W$, etc. in terms of $\partial W/\partial x$, $\partial W/\partial y$, etc. it follows that

$$h_1{}^2 = \frac{1}{v^2};\quad h_2{}^2 = \frac{1}{i^2}. \qquad \ldots\ldots(10)$$

The operator $\nabla^2$ can then be written

$$\begin{aligned}\nabla^2 &= \frac{1}{h_1 h_2}\left[\frac{\partial}{\partial W}\frac{h_2}{h_1}\frac{\partial}{\partial W} + \frac{\partial}{\partial \psi}\frac{h_1}{h_2}\frac{\partial}{\partial \psi}\right]\\ &= \frac{i^2}{\rho}\left[\frac{\partial}{\partial W}\frac{1}{\rho}\frac{\partial}{\partial W} + \frac{\partial}{\partial \psi}\rho\frac{\partial}{\partial \psi}\right] \qquad \ldots\ldots(11)\end{aligned}$$

and the well-known relationship between force and acceleration components perpendicular to a trajectory is

$$i\frac{\partial V}{\partial \psi} = -\frac{2V}{R_\psi} \qquad \ldots\ldots(12)$$

where $R_\psi$ is the radius of curvature of the trajectory.

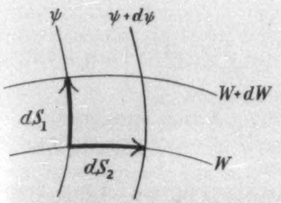

Figure 1. Elementary cell defined by the orthogonal curvilinear coordinates ($W$, $\psi$).

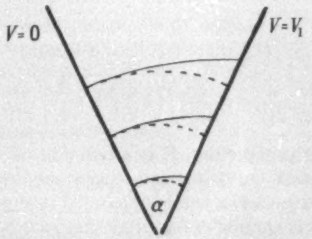

Figure 2. Electron trajectories between two inclined planes. Lines of force are arcs of circles shown by the broken lines.

Three general deductions regarding two-dimensional motion will now be made. For clarity they are stated as propositions.

### (i) *Proposition on Rectilinear Motion*

There are only two possible types of rectilinear motion in two dimensions, namely (*a*) that in which stream lines are parallel, and (*b*) that in which stream lines radiate from a point.

For lines of flow to be of zero curvature they must coincide with lines of force and hence (by (12)) $V$ and $v$ are functions of $W$ only. Equipotential lines coincide with lines of constant action and so have the same radius of curvature $R$ at any given point. These lines form a system of parallel curves and from (8) and (10) it follows that

$$\frac{\partial R}{\partial W}=\frac{1}{v}. \qquad \text{......(13)}$$

By means of (11)

$$\nabla^2 W=\frac{i^2}{\rho}\frac{\partial}{\partial W}\frac{1}{\rho}=-\frac{v^2}{\rho}\frac{\partial \rho}{\partial W}. \qquad \text{......(14)}$$

The operator $\nabla^2$ can also be written

$$\nabla^2=\frac{\partial^2}{\partial n^2}+\frac{1}{R}\frac{\partial}{\partial n} \qquad \text{......(15)}$$

where $\partial/\partial n$ indicates differentiation in a direction normal to a level line of the function operated on and $R$ is the radius of curvature of the level line. Thus

$$\nabla^2 W=\frac{\partial^2 W}{\partial n^2}+\frac{1}{R}\frac{\partial W}{\partial n}=v\left(\frac{dv}{dW}+\frac{1}{R}\right). \qquad \text{......(16)}$$

By combining (14) and (16) it follows that

$$\rho\left(\frac{dv}{dW}+\frac{1}{R}\right)+v\frac{\partial \rho}{\partial W}=0. \qquad \text{......(17)}$$

This equation must be satisfied identically. In addition Poisson's equation (2) must also be satisfied, which by means of (15) can be written

$$4\pi\rho=\nabla^2 V=v\left[\frac{d}{\partial W}\left(v\frac{dV}{dW}\right)+\frac{1}{R}\frac{dV}{dW}\right]. \qquad \text{......(18)}$$

It is now possible to eliminate $\rho$ between (17) and (18), and replacing $\partial R/\partial W$ by $1/v$, cf. (13), there results the identity

$$\left(2\frac{dv}{dW}\frac{d}{dW}+v\frac{d^2}{dW^2}\right)\left(v\frac{dV}{dW}\right)+\frac{1}{R}\left[\frac{dv}{dW}\frac{dV}{dW}+2\frac{d}{dW}\left(v\frac{dV}{dW}\right)\right]=0. \qquad \text{......(19)}$$

Clearly, therefore, $R$ is a function of $W$ only. Action lines are thus lines of constant curvature and so can only be a family of concentric circles, or in the limiting case, a family of parallel straight lines.

For simplicity this proposition has been stated and proved for two-dimensional motion. In the case of three-dimensional motion action surfaces coincide with equipotential surfaces and form a system of parallel surfaces. By a similar argument to the foregoing it can be shown, by investigating the principal radii of curvature of these surfaces, that there are only three types of rectilinear motion, namely lines of flow (*a*) parallel, (*b*) radiating from a point, and (*c*) radiating normally from an axis.

### (ii) *Proposition on Motion with Constant Current Density*

The only motions in which the current density is constant along a line of flow are (*a*) stream lines which are parallel straight lines, and (*b*) stream lines which are concentric circles.

The motion is characterized by the fact that $|\mathbf{i}|$ is a function of $\psi$ only. Clearly therefore, since $h_2 = 1/|\mathbf{i}|$ stream lines must form a family of parallel curves. Thus

$$\frac{\partial R_\psi}{\partial \psi} = h_2 = \frac{1}{|\mathbf{i}|} \qquad \ldots\ldots(20)$$

and hence on integration the most general form of $R_\psi$ is

$$R_\psi = \int \frac{1}{|\mathbf{i}|} d\psi + a \qquad \ldots\ldots(21)$$

where $a$ is a function of $W$ only.
If (21) is substituted in (12) there results

$$\frac{1}{2}\frac{\partial \log V}{\partial \psi} = -\frac{1/|i|}{\int(1/|i|)d\psi + a} \qquad \ldots\ldots(22)$$

and hence by integration and rearrangement

$$V^{1/2} = \frac{b}{c+a} \qquad \ldots\ldots(23)$$

where $b$ is a function of $W$ only and $c = \int(1/|i|)d\psi$ is a function of $\psi$ only. From (6), (9) and (23) it follows that the most general form of $\rho$ is

$$\rho = de + fg \qquad \ldots\ldots(24)$$

where $d$ and $f$ are functions of $W$ only, and $e$ and $g$ are functions of $\psi$ only.

If Poisson's equation (2) is now set down with the operator $\nabla^2$ in the form (11) it will be seen to consist of a number of terms each of which is the product of a function of $\psi$ only and a function of $W$ only. It is easy to show that this equation can be satisfied identically only under the alternatives (*a*) that all the functions of $W$ vanish or are constants, or (*b*) that all the functions of $\psi$ vanish or are constants.

Under the first alternative it follows from (21) that $R_\psi$ is a function of $\psi$ only, but the $\psi$-curves are a family of parallel curves and hence must consist of a family of concentric circles. A complete solution can easily be obtained but the case is of little practical importance. Under the second alternative the current density is everywhere constant, and from (21) $R_\psi$ is a function of $W$ only. Since the $\psi$-curves are parallel curves they must in fact be a family of straight lines.

### (iii) *Proposition on Harmonic Functions*

In a two-dimensional electron stream lines of flow cannot be represented by the level lines of a harmonic function except in trivial cases to be named.

Let $w$ be a harmonic function whose level lines represent lines of flow, and let $u$ be the conjugate of $w$. Clearly $\psi$ is a function of $w$ only, and $W$ is a function of $u$ only. By means of equations (7) it follows that

$$\rho = \frac{\psi'}{W'} \qquad \ldots\ldots(25)$$

and from (6)

$$V = \frac{m}{2e}[W']^2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2\right]. \qquad \ldots\ldots(26)$$

Since $u$ is harmonic it can be expressed as the sum of a complex analytic function and its conjugate, i.e.

$$u = f + f^* \qquad \ldots\ldots(27)$$

where $f$ is a function of $x+iy$ and $f^*$ is the same function of $x-iy$. Likewise

$$w=-i(f-f^*). \qquad \ldots\ldots(28)$$

Let

$$z=(m/2e)(W')^2 \qquad \ldots\ldots(29)$$

then equation (26) can be written

$$V=4zf'f'^* \qquad \ldots\ldots(30)$$

and hence

$$\nabla^2 V=16[(f'f'^*)^2 z''+\{(f')^2 f''^*+f''(f'^*)^2\}z'+f''f''^* z]. \qquad \ldots\ldots(31)$$

We must now consider the conditions under which Poisson's equation (2) can be satisfied when $\nabla^2 V$ is given by (31) and $\rho$ by (25). It is convenient to discuss three cases separately, corresponding to the following assumptions about $\rho$, namely (i) that $\rho$ is zero, (ii) that $\rho$ is a function of $u$ only, i.e. that $\psi'$ is a constant, and (iii) that neither $\psi'$ nor $W'$ are constants.

*Case* 1. $\rho=0$.

Since the current density must also be zero it follows that div $\mathbf{i}=0$. Poisson's equation reduces to the Laplace equation $\nabla^2 V=0$ and hence from (31) the condition to be satisfied is

$$(f'f'^*)^2\left[z''+\left\{\frac{f''}{(f')^2}+\frac{f''^*}{(f'^*)^2}\right\}z'+\frac{f''}{(f')^2}\cdot\frac{f''^*}{(f'^*)^2}z\right]\equiv 0. \qquad \ldots\ldots(32)$$

Now, if $f'f'^*$ is zero, $V$ is also zero by (30) and hence motion is impossible. We must therefore examine the conditions under which the expression contained in the square brackets vanishes.

If $z$ is zero, motion is impossible, and if $z$ is a constant there can be no acceleration in the motion, a trivial case. Let us assume therefore that both $z$ and $z'$ exist and proceed to examine their coefficients. The identity can clearly be established if either $f''$ is zero, or $f''/f'^2$ is a constant and it can easily be verified that the former refers to a motion in which the acceleration is constant and in a fixed direction, whereas the latter refers to a motion along lines radiating from a point (in fact the only possible cases of rectilinear motion).

If, however, the coefficients of $z$ and $z'$ do not vanish and are not constants we must examine the possibility that they are both functions of $u$ only. Now, the coefficient of $z'$ is real and is a harmonic function (since it is the sum of two complex conjugates) and hence it can only have the form $au$ where $a$ is a constant. This follows from the fact that the only function of a harmonic function which is itself a harmonic function is a constant multiplied by the original harmonic function. But if

$$\frac{f''}{(f')^2}+\frac{f''^*}{(f'^*)^2}=au$$

the coefficient of $z$, namely

$$\frac{f''}{(f')^2}\frac{f''^*}{(f'^*)^2}=\tfrac{1}{4}a^2(u^2+w^2)$$

is not a function of $u$ only and hence the identity cannot be established.

*Case* 2. ***$\rho$ a function of $u$ only.***

From (25) it follows that

$$\rho = \frac{b}{W'} = \frac{4c}{\pi z^{1/2}}$$

where $b$ and $c$ are constants, and hence combining equations (6), (2) and (31) the following identity must hold

$$(f'f'^*)^2 \left[ z'' + \left\{ \frac{f''}{(f')^2} + \frac{f''^*}{(f'^*)^2} \right\} z' + \frac{f''}{(f')^2} \frac{f''^*}{(f'^*)^2} z \right] \equiv -\frac{c}{z^{1/2}}. \qquad \ldots\ldots(33)$$

This identity may be discussed in the same way as the previous one. For example :

(i) If $z=0$ motion is impossible.

(ii) If $z$ is a constant, $f''$ and $f''^*$ must also be constants and it is easy to show that in the motion lines of flow are hyperbolae. It is important to note that in this case the action function is a harmonic function and the space charge density is constant. This case will be referred to later.

(iii) If $z$ and $z'$ exist then either $f''=0$, in which case lines of flow are parallel, or $f''/f'^2$ is a constant, in which case lines of flow are straight lines radiating from a point.

By carrying out a similar argument as in case 1 it can be shown that there are no other possible motions.

*Case* 3. *$\psi'$ and $W'$ exist and are not constants*

The identity to be established differs from (33) only in that $c$ is now a function of $w$ only. By multiplying both sides by $z^{1/2}$ it is clear that a necessary condition is that

$$z^{1/2}(f'f'^*)^2 \left[ z'' + \left\{ \frac{f''}{(f')^2} + \frac{f''^*}{(f'^*)^2} \right\} z' + \frac{f''}{(f')^2} \frac{f''^*}{(f'^*)^2} z \right]$$

should be function of $w$ only. That this is impossible can easily be shown by similar arguments to those set out in the previous cases.

To summarize, it can be stated that stream lines (and hence lines of constant action) can be represented by the plot of the level lines of harmonic function in only two types of motion, namely rectilinear motion and motion along hyperbolic paths. Three important corollaries can also be stated, viz.

*Corollary* 1. The only possible motion in which the space charge density is constant throughout is hyperbolic motion.

A necessary condition to be satisfied is

$$\operatorname{div} \mathbf{i} = 0 \qquad \ldots\ldots(3)$$

In irrotational flow the components of the vector current density $\mathbf{i}$ are $\rho \dfrac{\partial W}{\partial x}$ and $\rho \dfrac{\partial W}{\partial y}$ and hence (3) becomes

$$\rho \nabla^2 W + \operatorname{grad} \rho \operatorname{grad} W = 0. \qquad \ldots\ldots(34)$$

But if $\rho$ is a constant, grad $\rho$ is zero and hence $\nabla^2 W = 0$. The motion is thus according to Case 2 (ii) above.

*Corollary* 2. The only motion in which the space charge density is constant along a line of flow is hyperbolic motion.

This follows at once from Corollary 1. The restriction now imposed is that grad $\rho$ grad $W=0$ and hence by (34) $\nabla^2 W=0$.

*Corollary* 3. The only motions in which the space charge density is constant along lines of constant action are rectilinear and hyperbolic motions.

From equations (7) we have

$$\frac{\partial W}{\partial x}=\frac{1}{\rho}\frac{\partial \psi}{\partial y}; \quad \frac{\partial W}{\partial y}=-\frac{1}{\rho}\frac{\partial \psi}{\partial x}. \qquad \ldots\ldots(35)$$

By differentiating the first with respect to $y$, the second with respect to $x$ and subtracting there results

$$\frac{1}{\rho}\nabla^2\psi-\frac{1}{\rho^2}\operatorname{grad}\rho\operatorname{grad}\psi=0.$$

This is a necessary condition and in the present case there is the further restriction that grad $\rho$ grad $\psi=0$. It follows therefore that $\nabla^2\psi=0$; thus $\psi$ is a harmonic function and hence $\psi'$ must be a constant. The possible motions are thus described under Case 2 above.

## § 4. PARTICULAR SOLUTIONS

If attention is confined as in the foregoing to irrotational flow, the method suggested by Meltzer (1949) for finding particular solutions reduces to the choice for $W$, the action function, of an arbitrary polynomial or a set of simple known functions. It is unlikely that any new information can be obtained in this way since it is not enough merely to satisfy the field-dynamical equations. Thus, the case of hyperbolic motion given by Meltzer, which has been shown to be the only non-trivial case in which the action function is a harmonic function, namely $W=xy$, is not realizable since the conditions to be satisfied at a cathode are not fulfilled. It can be shown, also, that no real solutions can be obtained by expressing $W$ as a polynomial with a finite number of terms of the type $x^n y^m$ where $n$ and $m$ are integers (since when the velocity is zero the space charge density is finite). It would appear, therefore, that rigorous solutions are known only for the three types of rectilinear motion. There are, however, two cases of practical interest in which the motion is along curved paths and these will now be discussed.

## § 5. MOTIONS BETWEEN TWO INCLINED PLANAR ELECTRODES

The motion of electrons in a region bounded by two plane conductors inclined at an angle possesses some interesting features which do not appear to be known and which may have important applications. Consider first the case of negligible space charge and let it be assumed that the electrons depart from one plane with no initial velocity. With polar coordinates, as shown in Figure 2, the potential in the field can be written $V=V_1\theta/\alpha$ and equation (6) takes the form

$$\left(\frac{\partial W}{\partial r}\right)^2+\left(\frac{1}{r}\frac{\partial W}{\partial \theta}\right)^2=K\theta \qquad \ldots\ldots(36)$$

where $K$ is a constant whose value is $2eV_1/m\alpha$. Since the squared terms must be independent of $r$ we can seek a solution in the form $W=r\,y(\theta)$. Equation (36) then reduces to

$$y^2+y'^2=K\theta \qquad \ldots\ldots(37)$$

which is solved by

$$y=\sum_{n=1}^{\infty} a_n\theta^{(4n-1)/2}$$

The constants $a_n$ can readily be evaluated, for example

$$a_1 = \frac{2}{3} K^{1/2}, \qquad a_2 = -\frac{2}{21} a_1, \qquad a_3 = \frac{10}{2079} a_1, \text{ etc.}$$

The radial and transverse velocity components are given by

$$\dot{r} = \frac{\partial W}{\partial r} = y; \quad r\dot{\theta} = \frac{1}{r}\frac{\partial W}{\partial \theta} = y'$$

Hence

$$\frac{dr}{r} = \frac{y}{y'}\, d\theta$$

and by integration

$$r = r_0 \exp \phi(\theta) \qquad \text{......(38)}$$

where

$$\phi(\theta) = \int_0^\theta \frac{y}{y'}\, d\theta \,.$$

A series expansion for $\phi(\theta)$ can easily be obtained, namely

$$\phi(\theta) = \sum_{n-1}^{\infty} b_n \theta^{2n}$$

where

$$b_1 = \frac{1}{3}, \qquad b_2 = \frac{4}{189}, \qquad b_3 = \frac{32}{18711}, \text{ etc.}$$

Clearly the motion is such that the radii of the end points of the trajectories bear the same ratio to each other as do the starting points and so a perfect linear magnification is obtained. The time of transit of the electrons also has an interesting property To examine this consider the relationship between transverse acceleration and force in a trajectory, namely

$$m \frac{1}{r} \frac{d}{dt} (r^2 \dot{\theta}) = \frac{eV_1}{r\alpha}\,.$$

On removing $1/r$ from both sides and integrating with respect to time we obtain $t = 2r^2\dot{\theta}/K$ where $t$ is measured from the instant the electron leaves the first plane. Noting that $r\dot{\theta} = y'$, a function of $\theta$ only, it follows that

$$t = r\left(\frac{2}{K} y'\right). \qquad \text{......(39)}$$

Hence the transit time is proportional to the final (or starting) radius.

The above-mentioned properties of the motion both derive from the functional form of the action function. It is interesting to see, therefore, if by writing $W = rg(\theta)$ a possible motion can be defined in which the space charge is not negligible. By substitution in (6)

$$g^2 + g'^2 = \frac{2e}{m} V \qquad \text{......(40)}$$

and hence $V$ is a function of $\theta$ only.
By expressing the operator $\nabla^2$ in terms of polar coordinates and applying Poisson's equation (2) it follows that

$$\rho = \frac{V''}{4\pi r^2}. \qquad \text{......(41)}$$

The radial and transverse components of the vector current density are $i_r = V''g/4\pi r^2$ and $i_\theta = V''g'/4\pi r^2$.

Also
$$\operatorname{div}\mathbf{i} = \frac{\partial i_r}{\partial r} + \frac{1}{r}\,i_r + \frac{1}{r}\frac{\partial i_\theta}{\partial \theta}$$
$$= \frac{1}{4\pi r^3}\left[-2V''g + V''g + \frac{d}{d\theta}(V''g')\right].$$

Therefore, div **i** is zero if
$$\frac{d}{d\theta}(V''g') - V''g = 0. \qquad \ldots\ldots(42)$$

It is now possible to eliminate $V$ between equations (40) and (42) and obtain an ordinary differential equation in $g$. Any solution of this equation defines a possible motion. By writing
$$g = \sum_{n=1}^{\infty} \alpha_n \theta^{(6n-1)/3}$$
it is easy to show that the constants $\alpha_n$ can be determined such that (40) and (42) are satisfied. Corresponding to this value of $g$ it follows from (40) that $V$ can be expanded in a series of the form
$$V = \sum_{n=1}^{\infty} \beta_n \theta^{(6n-2)/3}.$$

Clearly, therefore, the potential gradient at the cathode (where $\theta = 0$) is zero and hence this solution gives the space-charge-saturated case. Proceeding as before it can be shown that the equation of a trajectory is now
$$r = r_0 \exp\left(\frac{3}{10}\theta^2 + \frac{3}{220}\theta^4 + \ldots\right). \qquad \ldots\ldots(43)$$

For small values of $\theta$ the magnification is thus only slightly less than in the space-charge-free case. The degree of magnification is quite small for angles less than $\pi/2$, less than three times, but for angles between $\pi/2$ and $\pi$ it increases rapidly to many thousands of times.

## §6. MOTION BETWEEN TWO COAXIAL RIGHT CIRCULAR CONICAL ELECTRODES WHOSE VERTICES COINCIDE

Consider first the space-charge-free case. Let $(r, \theta, \phi)$ be spherical polar coordinates and let the axis of the cones be the line $\theta = 0$. It is easy to show that the potential in the field between the cones can be written
$$V = a \ln \tan \tfrac{1}{2}\theta + b \qquad \ldots\ldots(44)$$
where $a$ and $b$ are constants depending on the vertical angles and potentials of the cones. Clearly a possible form of the action function is $W = rg(\theta)$ and, as before,
$$g^2 + g'^2 = \frac{2e}{m}V. \qquad \ldots\ldots(45)$$

The motion is confined to azimuthal planes and the equation to a trajectory has the form $r = r_0 F(\theta)$.

If the conical electrodes have the same vertical angle an extremely important property is to be noted, namely that the arrival pattern of the electrons on the anode is a perfect magnification of the starting pattern at the cathode. This fact may be proved quite simply, for example by developing the two conical surfaces into a plane.

The integration of (45) with $V$ as in (44) can be effected by a variety of means. For example, if $\alpha$ is the vertical angle of the first cone and if $t=\tan\frac{1}{2}(\theta-\alpha)$ and $T=\tan\frac{1}{2}\alpha$, a solution valid for $t<T$ can be obtained by expanding $V$ as a power series in $t$ and writing

$$g(\theta)=\sum_{n=1}^{\infty} a_n t^{(2n+1)/3}.$$

Proceeding in this way the equation to a trajectory becomes

$$r=r_0\exp\frac{4}{3}t^2\left[1+\frac{1}{15}\left(\frac{1}{T}-T\right)t-\frac{1}{21}\left\{\frac{32}{3}+\frac{11}{20}\left(\frac{1}{T}-T\right)^2\right\}t^2+\dots\right]. \qquad \dots\dots(46)$$

The property of perfect magnification also exists when the region between the cones is space-charge-saturated. This can be shown by the same argument as was applied in discussing motion between two inclined planes.

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# A 100-Kilowatt Water-cooled Solenoid

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*ABSTRACT*. Design data and constructional details are given for a small water-cooled solenoid, which, for a consumption of 100 kw. produces a field of 14·7 kilogauss, uniform to within $\pm\frac{1}{2}\%$ in a cylindrical volume 6 cm. long and 4 cm. diameter. The solenoid contains no iron or other ferromagnetic material, and is in use at the Clarendon Laboratory, Oxford, for experiments on adiabatic demagnetization.

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## § 1. INTRODUCTION

We required for experiments on adiabatic demagnetization a solenoid capable of producing a strong magnetic field, if possible greater than 10 kilogauss. The nature of the experiments to be performed with it required that the field should be uniform to within $\pm\frac{1}{2}\%$ over a region some 6 cm. in length, that the solenoid should contain no iron or other ferromagnetic material, that the inside diameter should not be less than 57 mm. so that it would fit easily round a glass Dewar vessel of the type in use for these experiments, and that it should not be too bulky, and fairly easy to construct. We had at our disposal a D.C. generator which would deliver 300 amp. at 330 v. The solenoid to be described is one made to suit these requirements.

## § 2. GENERAL CONSIDERATIONS

The basic theory of high-powered air core solenoids was worked out long ago. Maxwell (1881) showed that, for a given available electric power, the greatest field obtainable from a uniformly wound coil is obtained when a longitudinal section through the coil, containing the axis of symmetry, exhibits the profile $r^2=\sin\theta$. The line $\theta=0$ is the axis of symmetry. This shape is useless for our purpose as the windings extend right to the origin, and it would be difficult to construct.

It has been shown that for coils with a rectangular section winding space (Fabry 1898, Cockcroft 1928) the field $H$ at the centre is given by the formula

$$H=G\left(\frac{W\eta}{a_1\rho}\right)^{1/2}, \qquad \ldots\ldots(1)$$

where $W$ is the power dissipated in the coil, $a_1$ is the inner radius of the winding space, $\rho$ is the resistivity of the winding material, $G$ is a dimensionless factor which depends only on the shape of the winding space, and $\eta$ is the 'filling factor' viz.

$$\frac{\text{volume of winding space occupied by conductor}}{\text{total volume of winding space}}.$$

Fabry and Cockcroft both use a mixed system of units, expressing $W$ in watts, $a_1$ in centimetres, $\rho$ in ohm cm., and $H$ in gauss. With these units the maximum value of $G$ is 0·179 and for this the outer radius of the winding space is $3a_1$ and the axial length of the winding space $4a_1$.

Bitter (1936) considers the case of solenoids of other shapes than with rectangular section winding space and also with non-uniform windings. In this case equation (1) still holds. Values of $G$ are given for these types of coil, many of which would be very difficult to construct. He gives, for the most efficient coil with any current distribution, $G = 0{\cdot}272$. This coil extends to infinity in all directions. The best value of $G$ quoted for any finite coil is a little greater than 0·2; coils with rectangular winding space and uniform winding do not compare unfavourably with these fancy-shaped coils.

A simple calculation shows that, for $a_1 = 3$ cm., about 50 kw. is required to produce 10 kG. in the most favourably shaped coil of rectangular section winding space. The heat so produced must be removed continuously; many methods have been suggested for cooling coils of this type, and are reviewed by Bitter (1936), Gaume (1947) and others. We decided to use a design originally due to Tsai (1947). Bare copper strip is used for the windings and adjacent turns are separated from each other by winding round the strip a nylon filament about 0·3 mm. diameter, with a pitch of 3 mm. Ordinary tap water cannot be used as a cooling agent, as electrolysis takes place which rapidly corrodes the windings. Distilled water is therefore used, and cooled by tap water in a heat exchanger. This method of construction permits filling factors as high as 0·7, with high current densities and efficient cooling while requiring only small pressures to pass the water through sufficiently rapidly.

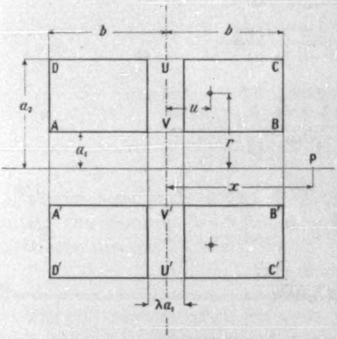

Figure 1. Longitudinal section through windings.

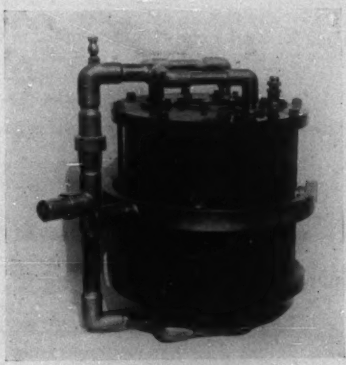

Figure 2. Photograph of the solenoid.

## § 3. DESIGN OF THE SOLENOID

In the first place it follows from Maxwell's electromagnetic equations that, if the field is uniform along the axis of the solenoid, it is also uniform away from this axis. It is, therefore, only necessary to ensure that the field is uniform along the axis. Now the very efficient coils mentioned in the previous section do not produce a very homogeneous field. For example, for the most efficient solenoid with rectangular winding space, the axial field has fallen by 1% at a distance $0{\cdot}52a_1$ from the centre.

We decided to use a solenoid with rectangular section winding space, with length $6a_1$ and outer radius $3a_1$, and to omit from the centre of the coil a slice making a system analogous to the Helmholtz double coil (Figure 1). This was found to give a sufficiently uniform field over the required volume: the value of $G$ is 0·159, which is only 11% less than the theoretical maximum for a coil with rectangular section winding space and with no gap in the middle.

In order to calculate the shape of the winding space, it seemed to us an appropriate approximation (mainly because without it the calculations would have been far too tedious) to treat the windings as giving a uniform current density $\tau$ E.M.U/cm² flowing everywhere in the rectangle ABCD in Figure 1 and to account for the slice omitted by superimposing a surface current density $\lambda a_1 \tau$ E.M.U/cm. circulating in the plane annulus UV in a direction opposed to the direction of the main current. Thus $\lambda a_1$ is the width of the gap.

If we consider the field at a point P on the axis of a circular current, radius $r$ (Figure 1), the field is

$$\frac{2\pi r^2 i}{\{r^2+(x-u)^2\}^{3/2}}.$$

Then for a solenoid with rectangular winding space

$$H_1=\int_{u=-b}^{+b}\int_{r=a_1}^{a_2}\frac{2\pi r^2\tau\,dr\,du}{\{r^2+(x-u)^2\}^{3/2}}.$$

If we evaluate this and put $a_2=\alpha a_1$, $b=\beta a_1$, $x=\xi a_1$, then

$$H_1=2\pi\tau a_1\left[(\beta-\xi)\left\{\sinh^{-1}\frac{\alpha}{|\beta-\xi|}-\sinh^{-1}\frac{1}{|\beta-\xi|}\right\}\right.$$
$$\left.+(\beta+\xi)\left\{\sinh^{-1}\frac{\alpha}{\beta+\xi}-\sinh^{-1}\frac{1}{\beta+\xi}\right\}\right]. \qquad \ldots\ldots(2)$$

Similarly, the flat coil lying in the plane UU′ will produce a field

$$H_2=\int_{a_1}^{a_2}\frac{2\pi r^2(-\lambda a_1\tau)\,dr}{(r^2+x^2)^{3/2}}.$$

Therefore

$$H_2=-2\pi\tau a_1\lambda\left[\sinh^{-1}\frac{\alpha}{\xi}-\sinh^{-1}\frac{1}{\xi}-\frac{\alpha}{(\alpha^2+\xi^2)^{1/2}}+\frac{1}{(1+\xi^2)^{1/2}}\right]. \qquad \ldots\ldots(3)$$

If $\xi$ is small, we get

$$H_2\simeq-2\pi\tau a_1\lambda\left[\ln\alpha-\frac{3}{4}\xi^2\left(1-\frac{1}{\alpha^2}\right)+O(\xi^4)\right]. \qquad \ldots\ldots(4)$$

The total field $H=H_1+H_2$.

The axial field can be expressed as a Taylor series in terms of $\xi$ and, by symmetry, only even powers of $\xi$ will be present. In order to make the field uniform, we must arrange that as many as possible of the first terms in this expansion vanish.

Now

$$\left(\frac{d^2H_1}{d\xi^2}\right)_{\xi=0}=4\pi\tau a_1\left[\frac{\beta^{-4}}{(1+\beta^{-2})^{3/2}}-\frac{\alpha^3\beta^{-4}}{(1+\alpha^2\beta^{-2})^{3/2}}\right]. \qquad \ldots\ldots(5)$$

This is always negative for a finite solenoid with no gap, so we must choose $\lambda$ so that $(d^2H/d\xi^2)_{\xi=0}=0$.

Now, from (4), $(d^2H_2/d\xi^2)_{\xi=0}=3\pi\tau a_1\lambda(1-\alpha^{-2})$ so that

$$\left(\frac{d^2H}{d\xi^2}\right)_{\xi=0}\propto 4\left[\frac{\beta^{-4}}{(1+\beta^{-2})^{3/2}}-\frac{\alpha^3\beta^{-4}}{(1+\alpha^2\beta^{-2})^{3/2}}\right]+3\lambda(1-\alpha^{-2})=0.$$

This equation gives $\lambda$ in terms of the known $\alpha$ and $\beta$.

If we desired, we could arrange two more slices equal in size and symmetrically placed about UU′ to make $(d^4H/d\xi^4)_{\xi=0}$ zero also, and could continue this process of making further Taylor coefficients vanish.

In our case, where $\alpha=3$ and $\beta=3$, we find that $\lambda=0{\cdot}180$, justifying the assumption made at the beginning of this section that the slice omitted is thin. For the coil constructed, we put $\lambda=0{\cdot}25$. This gives two small humps in the graph of $H$ against $\xi$, one each side of the origin, and further extends the region in which $H$ is constant to within ± ½% (see Figure 3).

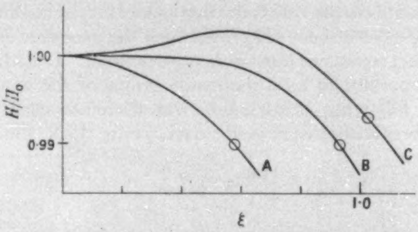

Figure 3. $H/H_0$ as a function of $\xi$. A: for a coil $\alpha=3$, $\beta=3$, no gap; B: $\alpha=3$, $\beta=3$, $\lambda=0{\cdot}182$; C: $\alpha=3$, $\beta=3$, $\lambda=0{\cdot}250$. The circles denote the end of the useful region, i.e. where the total variation in $H$ is less than 1% of $H_0$.

The final step in the design is to choose the copper strip to give a total resistance equal to the optimum load resistance of the generator which will supply the power; in our case this was 1·1 ohms.

## § 4. CONSTRUCTION

The coil consists of 20 flat coils, or 'pancakes', wound with copper strip ¼in. × 20 S.W.G. The inner end of the strip was soldered on to a thin brass tube 61·5 mm. outside diameter, ¼ in. long, and ½ mm. thick. The solder used was Johnson Matthey's L. M. 10. Nylon thread 0·3 mm. diameter was wound round the copper strip by means of the winding device shown in Figure 4.

The tube A which carries a cheek on the left is a push fit into the ball race B and into the sprocket S, and is turned by an electric motor, which drives S by means of a chain. The ball race B is clamped into a thick aluminium bracket C. D is a bobbin which rotates freely on A, and on to which the nylon filament is first wound. The copper strip XX passes along the axis of A, and moves from right to left during the winding. It is located centrally by passing through a slot in the plate E, which is held loosely in a circular groove in the end cheek of A, so that E stays still as A rotates. The end cheek of A also carries an arm, on the end of which is a wheel F. Nylon filament from the bobbin D passes over the wheel F and thence round the copper strip. The copper strip was fed into the device at a speed of about 2 ft/min. from a pair of rollers, driven from the same motor as that which drives the sprocket S. By a suitable choice of reduction gear between the motor and the rollers, the pitch of the nylon winding could be adjusted to the desired value. This winder was designed and made by one of our mechanics, Mr. J. J. Milligan, who also attended to many of the finer details, not mentioned here, which ensure its smooth working.

The distance between consecutive turns of nylon was about $2\frac{1}{2}$ mm. The brass tube was slipped on a bush between two cheeks $\frac{1}{4}$ in. + 0·6 mm. apart, and the pancake was wound at the same time as the nylon was wound on the strip. When the required outer diameter for the pancake, 181·5 mm., was reached, the copper strip was severed and the pancake temporarily secured by wrapping insulating tape round its circumference. The pancakes were insulated from each other by means of wheels cut from tufnol or paxolin sheet, with 8 spokes, 2 mm. wide, shaped so as to impede the water flow as little as possible. These wheels were cut from $1\frac{1}{2}$ mm. sheet, except the specially thick wheels used in the centre to provide the gap in the winding, and at the ends to provide collecting spaces for the cooling water.

When designing electrical conditions between the pancakes, it must be remembered that the contact resistance must be kept very small. In addition, equation (1) shows that it is important to have the inside radius of the windings as small as possible, and the following construction was therefore used. For the inside connections, thin brass tubes were made, each $\frac{1}{2}$ mm. thick, $\frac{1}{2}$ in. long, with a stiff push fit into the tubes on which the pancakes were wound. The pancakes were placed together in pairs, with a paxolin wheel between each, the diameter of these wheels being just less than the inner diameter of the outer case. The electrical and mechanical connection between the two pancakes was made by pressing one of these brass tubes into the hole in the centre. The inside of the magnet case was a brass cylinder, 59 mm. outside diameter, 16 S.W.G. thick and 22 cm. long. A few layers of polythene* sheet were wrapped round this, and the pairs of pancakes were slipped on with a paxolin wheel between each pair, the outer diameter of these wheels being just less than the outer diameter of the pancakes. Electrical connection was made between adjacent pairs of pancakes on the outside, by removing the temporary binding of insulating tape, and winding a large copper band round both adjacent pancakes. The ends of the band were secured by two screw clips, one

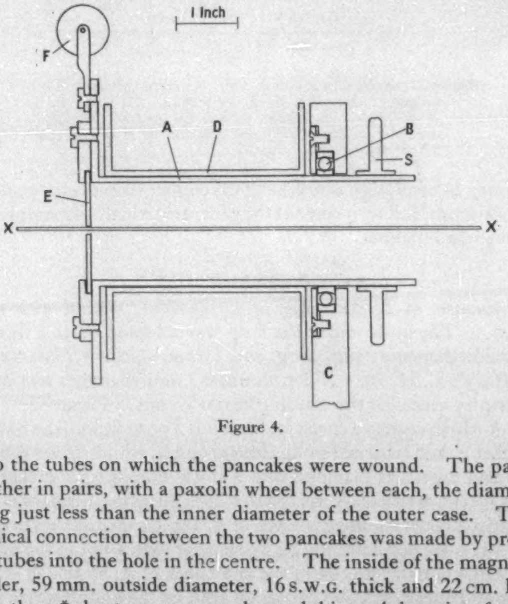

Figure 4.

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* Originally, a sheet of resin-bonded cloth was used, but this broke down and a spark passed between the windings and the case. This occurred after a few weeks' use, and is believed to have been due to water entering the material via the ends of the cloth fibres.

bearing down on each pancake. Special bands were used for the centre and for the ends, and the terminals were soldered to the bands securing the end pancakes.

The outer cylindrical case was a copper cylinder 8 in. outside diameter, and 10 S.W.G. thick. The large paxolin wheels have an outside diameter which allows them to slide into this, leaving just sufficient room for a sheet of paxolin paper between their edges and the outer case. The end cheeks are made of tufnol $\frac{5}{8}$ in. thick. These are held together by eight long brass rods, $\frac{1}{4}$ in. diameter tapped 0 BA at the ends. The joins between the end cheeks and the rest of the case, and where terminals and water-pipes pass through the end cheeks, are sealed by means of 'Gaco' rubber rings, in seals whose construction is obvious from the diagram (Figure 5). The cooling water enters at three points spaced 120° apart at the middle of the copper outer case and leaves at three equally spaced points on each end cheek. Holes are cut in the paxolin paper lining of the case, and in the central copper connecting band, to let the water through. An air release valve of the type used on radiators for central-heating systems is provided to ensure that no air is trapped in the solenoid.

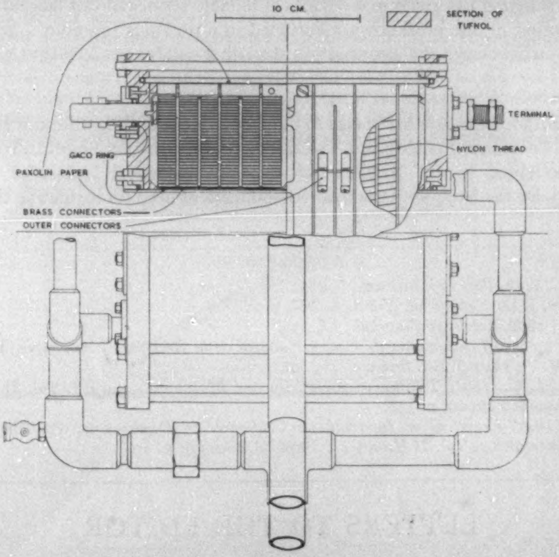

Figure 5. Section and elevations showing construction of the solenoid. For simplicity in this diagram the solenoid is shown as being composed of 12 pancakes.

## § 5. PERFORMANCE

The cooling water is circulated by a 'Safran' centrifugal water-pump, made by the Saunders Valve Company, which delivers about 40 gal/min. against a pressure of 3 atmospheres. This water is cooled with tap water in a heat exchanger, similar to the one described by Hudson (1949). The field at the centre of the solenoid is linear with current, and with 280 amps a field of 14·7 kilogauss is obtained. The filling factor for the solenoid is 0·43, and it is easily verified that the field given by equation (1) agrees with the experimental value; with this current the power consumed is 99·5 kw. The resistance of the coil when cold is 1·16 ohms and the temperature rise of the cooling water on passing through the solenoid is about 8° c. The weight of the solenoid and case, without cooling water, is 67 lb.

The homogeneity of the field was measured, using two identical search coils connected in opposition, fixed on a rod so that their separation could be varied. When removed from the field, the throw of the ballistic galvanometer to which they were connected was proportional to the difference in the fields at the two coils. Within the limits of experimental error, the results are in agreement with the calculated values of the field.

This solenoid has given many hours of reliable service in this laboratory, with the exception of one breakdown mentioned in a previous footnote. Work is in progress on the design and construction of another coil for our 2,000 kw. generator.

### ACKNOWLEDGMENTS

The author expresses his thanks to the Department of Scientific and Industrial Research for a maintenance grant, and also to Dr. N. Kurti for helpful advice. He is also indebted to Dr. Swallow of Imperial Chemical Industries (Plastics Division), for the free gift of the nylon monofilament used for insulating the windings.

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# LETTERS TO THE EDITOR

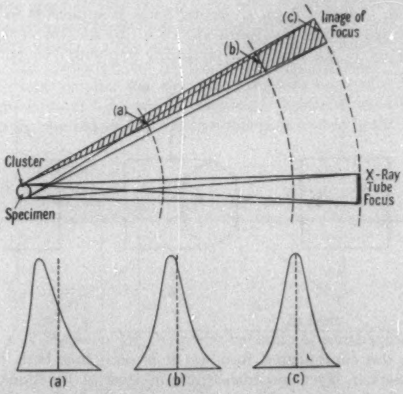
## Precision Lattice-Parameter Measurements

Recent measurements by the present author (Keith 1950) of the lattice-parameters of one natural and three synthetic samples of clear crystalline quartz have given results which differ from one another by more than the experimental error and which, except for one of the synthetic samples, are significantly smaller than the accurate values given by Lipson and Wilson (1941). The absolute precision of the measurements, as distinct from the precision relative to a specific camera calibration, was slightly uncertain, but believed to be of the order of 1 part in $10^5$. In view of the criticism based upon these results of the quartz calibration standard for x-ray powder cameras it seemed desirable to confirm this accuracy estimate, and hence the disagreement between the results and the values of Lipson and Wilson. This has now been done by comparing the parameters previously measured for a sample of Brazilian quartz with a 19 cm. 'Unicam' camera with those determined for the same sample with a 6·4 cm. Straumanis camera (Straumanis 1949) which has recently been constructed in this laboratory. Cameras of the Straumanis type are absolute instruments in which an asymmetrical film arrangement is employed to circumvent the necessity for preliminary calibrations. Copper K$\alpha$ radiation was again used, and the calculation of the quartz parameters from the measured Bragg angles for four separate films was carried out exactly as in the previous experiments. The results are compared below with those obtained earlier (values in kx.).

19 cm. ' Unicam ' camera (25° c.) $a = 4{\cdot}90309_3$; $c = 5{\cdot}39367_3$; $c/a = 1{\cdot}100055$.
6·4 cm. Straumanis camera (25° c.) $a = 4{\cdot}90314_4$; $c = 5{\cdot}39373_2$; $c/a = 1{\cdot}100056$.

It is seen that the two sets of values agree to within 1 part in $9 \times 10^4$, showing that the absolute precision of the earlier parameters is almost as great as was estimated. Comparison of values of the lattice-parameter of silver bromide measured with these two cameras has further confirmed the accuracy of the determinations carried out with the ' Unicam ' camera. It is interesting to note that Straumanis has obtained a consistent absolute precision of 1 part in $2 \times 10^5$ in measurements of the lattice-parameter of Al which he has carried out with his cameras.

Some observations which have been made during the investigation of quartz with the Straumanis camera are worthy of mention. With a specimen 0·25 mm. in diameter slightly spotty diffraction lines were obtained at all angles, and for the experiments a 0·5 mm. diameter specimen had to be used to ensure uniform diffraction lines. Even taking into account the small height (1 mm.) of the specimen being irradiated this behaviour is surprising, and is explained principally by the unfavourable ratio of the specimen–film and specimen–target distances which was used. The effect of varying this ratio is clarified by the accompanying diagram, showing in exaggerated form the influence on diffraction-line contours of a cluster of crystallites reflecting strongly at any given instant from one localized region of the specimen. Film position (a) corresponds to the situation in the Straumanis camera, and position (b) to that in the ' Unicam ' camera. A considerable improvement would be brought about by shortening the collimator of the Straumanis camera and advancing the specimen towards the target. Powder photographs of metal systems taken with this camera are generally free from spottiness, even with thin specimens, due to the high multiplicity factors of the reflecting planes for simple structures and the inherent diffuseness of the x-ray reflections from metallic crystals.



Diffraction-line contours at various film positions (see text).

In conclusion, it should be emphasized that the above effect may, in the case of crystals giving sharp x-ray reflections, invalidate the use of very thin specimens which are often desirable (Keith 1950). The unsuspected lack of uniformity which may occur in the line contours of apparently perfect diffraction lines would be most easily overlooked with the high measuring microscope magnifications also recommended previously. Provided that due care is taken and, in particular, that the ratio of the specimen–film and specimen–target distances is kept as near as possible to unity, the advantages of thin specimens and high magnifications are, nevertheless, considerable. It is conceivable that some of the parameters reported earlier for synthetic quartz may contain slight errors due to this effect and the high magnification which was used. The validity of the general conclusion regarding the reliability of camera calibrations by the quartz method cannot, however, be affected.

H. H. Wills Physical Laboratory,
University of Bristol.
4th September 1950.

H. D. KEITH.

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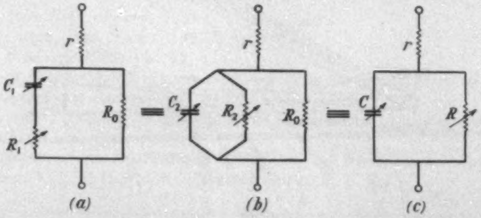
# The Capacitance of Selenium Rectifiers

In his recent paper R. Cooper (1950) suggested that the observed variation of impedance of a selenium rectifier in the audio-frequency range showed the presence of anomalous dispersion. However, from his Figure 3 (in which the ordinate axis should be labelled as reactance) this conclusion does not appear to be unequivocal since a good fit to *all* the points can be obtained by a circle whose centre does not lie on the resistance axis but at approximately (1300, −200). Such a circular locus is applicable to an impedance which can be represented in either of the forms shown in (*a*) and (*b*) in the diagram which are equivalent to the form (*c*), the symbol of variability implying frequency dependence. The circular locus for impedance is obtained when $\omega C_1R_1$ or $\omega C_2R_2$ is a constant independent of frequency. In the consolidated form (*c*) it is seen that both the components $C$, $R$ representing the barrier layer impedance are in general functions of frequency. The property of a circular locus is readily seen by noting that $C_1R_1$ or $C_2R_2$ in parallel with $R_0$ has a straight-line locus in the admittance plane which becomes a circle by inversion into the impedance plane; the resistance $r$ then displaces the circle by $r$ to the right in the impedance plane.



Furthermore, by applying the relationship between the real and imaginary parts of an impedance to $C_1R_1$ or $C_2R_2$ it is found that on writing

$$\omega C_1R_1 = \frac{1}{\omega C_2R_2} = \tan\frac{m\pi}{2} \qquad 0 \leqslant m < 1$$

the capacitances $C_1$ or $C_2$ must vary as $\omega^{-m}$ and the resistances $R_1$ or $R_2$ as $\omega^{-1+m}$.

This behaviour is of the type discussed by Fricke (1932) in connection with electrolytic polarization assuming a relaxation function of the form $t^{-m}$. It has been observed experimentally in selenium barrier-layer photocells by Wood (1933) who made impedance measurements on the audio-frequency range. It is not surprising that there should be an analogy between the time lag behind the applied field of the polarization and depolarization at an electrode in an electrolyte and of the ionization and de-ionization of impurity atoms in the barrier layer of a rectifier. In the case of selenium with a cadmium counter-electrode the activation energy is probably of the order of 0·1–0·2 ev., which is larger than $k\theta_D$ (where $\theta_D$ is the Debye temperature of selenium), and the theory of Goodman, Lawson and Schiff (1947) then indicates relaxation frequencies of the order of those observed by Cooper.

Radio Research Station,
Slough, Bucks.
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R. E. BURGESS.

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# The Correction of the Spherical Aberration of Electron Lenses using a Correcting Foil Element

Several systems for the correction of the spherical aberration of electron lenses have been proposed (Scherzer 1947, Gabor 1949, Hubert 1949, Ramberg 1949) but have all been shown to provoke practical difficulties which have precluded their use, with the possible exception of the crossed cylindrical lens system proposed by Scherzer.

In particular, it has been established that a departure from field continuity, by the introduction of conducting foils into the optically active region of an electron lens, may lead to a correction of the spherical aberration (Scherzer 1949). The performance of a suitable foil correction element, which is attractive because of its basic simplicity, has been examined in this laboratory. The characteristics of this element are outlined in the following.

Consider first a plane parallel foil element, with foils separated by a distance *d* and maintained at potentials $\phi_n$ and $\phi_1$, which produces a uniform retarding field in an intermediate region between the object $Q_n$ and the objective lens (Figure 1). In this field the radial component of velocity of an extra-axial electron trajectory $r_\alpha(z)$ is unaffected, whereas the axial component of velocity is diminished, and a divergence of the electron trajectory results. In addition, it is apparent that this divergence is less for a paraxial trajectory, because of the smaller radial component of velocity. Thus the foil element produces a negative spherical aberration which will correct the positive aberration of the following objective lens. This simple physical interpretation is confirmed by a rigorous analysis.

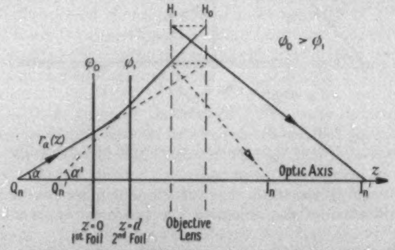

Figure 1. The uniform retarding field produced by a plane parallel foil element deviates an electron trajectory $r_\alpha(z)$ away from the axis, in such a way as to correct the spherical aberration of a succeeding objective lens, which is represented by its principal planes $H_0$ and $H_1$.

The trajectory of an electron, which originates with inclination α from an axial object point, which is taken in coincidence with the first foil, may be expressed

$$r(z) = -\frac{\sin 2\alpha}{2K}\left[1-\left(1+\frac{2Kz}{\cos^2\alpha}\right)^{1/2}\right],$$
$$K=(\phi_1-\phi_0)/2\phi_0 d.$$

It follows that the effective displacement of a 'marginal object point' is

$$\Delta Q_n = d - \frac{\cos^2\alpha}{K}\left(1+\frac{2Kd}{\cos^2\alpha}\right)^{1/2}\left[\left(1+\frac{2Kd}{\cos^2\alpha}\right)^{1/2}-1\right],$$

so that the resulting displacement between 'marginal' and 'paraxial' object points may be expressed, in third-order approximation,

$$\Delta_n = -\frac{d}{4}\frac{\Delta\phi}{\phi_0}\tan^2\alpha\left[1-\frac{1}{2}\frac{\Delta\phi}{\phi_0}\frac{(1+\cos^2\alpha)}{\cos^2\alpha}\right]$$

$$(\Delta\phi/\phi_0)^2<1, \qquad \Delta\phi=\phi_1-\phi_0.$$

Accordingly the condition for the correction of the third-order spherical aberration of the main lens becomes

$$\frac{\Delta\phi}{\phi_0} = -\frac{1}{2}\left[\left(1+\frac{16C_s}{d}\right)^{1/2}-1\right],$$

where $C_s$ is the spherical aberration constant (Glaser 1940 a).

An estimate of the practical requirements for correction is obtained by using values of the aberration characteristics $C_s/f$ for the axial magnetic lens distribution,

$$H(0, z)=H(0, 0)/[1+(z/a)^2],$$

tabulated by Glaser (1940 b). It is found that the correction of a typical lens requires a foil separation $d$ equal to the focal length $f$ of the main lens, with a potential increment $\Delta\phi=-0{\cdot}7\phi_0$. Thus the conditions for correction, as given by this simple treatment, appear not unreasonable.

Similar reasoning shows that a foil element which introduces an accelerating field after the main lens (Figure 2) will also correct the spherical aberration. However, in this case, the deviation of the imaging electrons that the element must produce is increased in proportion to the magnification of the objective lens, and the appropriate foil potentials are prohibitive.

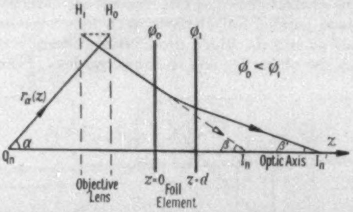

Figure 2. The accelerating foil element deviates an electron trajectory $r_\alpha(z)$ away from the axis in such a way as to correct the spherical aberration of a preceding electron lens.

The simple analysis given above serves to indicate the many problems arising in the practical design of a foil-corrected lens. The more important of these problems will now be mentioned.

In the corrected lens a part of the foil element will need to be located within the main lens, if a reasonable stage magnification is to be obtained. Thus a discrete treatment of the performance of the foil element is not fully comprehensive. In addition, the superposition of the lens and element fields will result in an electron retardation within the lens, with an accompanying modification of the electron-optical performance of the main lens, and it will be necessary to introduce overall changes in the main lens construction to allow for this effect.

Very thin beryllium foils can be obtained (Hast 1948) which may prove suitable for use in correcting the spherical aberration. However, it is probable that spurious effects, due to scattering of the electrons within the foil and to the charging of surface contamination, may invalidate the corrective properties of the foils. This difficulty may be surmounted arbitrarily. The object may be mounted on the first foil itself. Further, it is suggested that an apertured electrode may replace the second foil without appreciably disturbing the field configuration, provided that the aperture is small compared with the foil separation. However, a complete treatment of this problem will need to account for the finite field perturbation produced by the aperture, and must include the analysis of the superimposed action of the lens and element fields.

Further investigation of these problems is being continued in this laboratory.

The writer has recently been informed, through private communication, that Mr. M. E. Haine, of the Associated Electrical Industries Research Laboratories, has carried out a similar investigation in an analysis based on the analogous negative spherical aberration of a parallel-sided glass plate (Dyson 1949).

The writer wishes to express thanks to Professor J. Sayers for his encouragement throughout this work, and to Dr. G. Liebmann for his constructive criticism of this corrective system. This work is made possible by a grant from the Department of Scientific and Industrial Research.

Department of Electron Physics,
University of Birmingham.
U. F. GIANOLA.

MS. originally submitted 16th June 1950;
shortened to Letter 8th September 1950.

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## Aurora and Luminous Night Clouds

At about 2200 G.M.T. on 24th July 1950 there appeared together above the northern horizon in Central Scotland an auroral arc with summit base at an elevation of about 10° *and* luminous night (noctilucent) clouds reaching an elevation at 5° (Plate I). The simultaneous occurrence of these two phenomena must be very rare indeed. As twilight waned, the clouds extended steadily over the north-eastern sky and by 0050 G.M.T. on the 25th, they covered an area reaching along the horizon between azimuths 345° and 85° and upwards to a maximum elevation of nearly 25°. Meanwhile the aurora had vanished or had been extinguished by the brilliant light of the clouds. The wave structure, characteristic of these clouds, is shown in Plate II.

Using a Stormer–Krogness auroral camera, photographs showing cloud development were taken at regular intervals during the night until the clouds disappeared against the intensifying background of skylight at 0309 G.M.T. (sunrise 0402 G.M.T.). Unfortunately, photographs were obtained from only one station out of four normally used for simultaneous photography, but, as previous observation has shown that the clouds are always situated at about 85 km. (Jesse 1896, Störmer 1933 a, b, 1934, 1935 a, b) even single photographs allow their position, rate of movement and wave-crest separation to be measured if this altitude is assumed.

The constitution of luminous night clouds is uncertain. That their luminosity comes from sunlight scattered by the cloud particles is established, for their spectrum shows the Fraunhöfer lines but no emission lines, and the predominant colour is blue. (During this display, a change in colour of portions of the cloud from vivid blue to white was observed, suggesting growth of the particles. However, no such change had been observed during a fine though less extensive display on the night of 10–11 July 1949 (Paton 1949).) The appearance of the clouds always at a height of between 80 and 90 km. accords with the height distribution of temperature calculated by Whipple (1943) from meteor track photographs. These reveal a large lapse rate of temperature between a maximum of 350° K. at 60 km. and a minimum of about 200° K. below an inversion at 80 km.

Vegard (1935) has suggested that the clouds are formed by the condensation of water vapour on sodium oxide nuclei, formed by the combination of sodium and atmospheric oxygen. But it is difficult to explain how a supply of water vapour sufficient in quantity to form extensive clouds may exist at this great height, though Vegard proposes that it might be formed by the combination of atmospheric oxygen, perhaps in the form of ozone, and hydrogen, precipitated into the atmosphere from the sun. However at 80 km., where a temperature minimum exists and the pressure is of the same order of magnitude as the vapour pressure of water, conditions might conceivably be favourable for condensation.

The more general belief is that the clouds consist of residual meteoric or comet dust, which under the prevailing temperature distribution drifts to and remains below the base of the inversion at about 80 km. This is the view held by Professor Störmer who has witnessed and made photographic measurements of many displays.

Because the clouds appeared very frequently during the summers following the Krakatoa eruption in 1883 (the clouds are visible only during the summer months and between latitudes 45° and 62°), it was at first thought that they consisted of the products of condensation of gases projected to great heights during a volcanic eruption. But Vestine (1934) has pointed out that during the period 1880–1887 there occurred a remarkable number of meteor appearances, showers and great comets, and that this evidence, together with the fact that brilliant displays followed immediately after the great Siberian meteor of 1907 while no conspicuous occurrence succeeded the eruption of Katmai, Alaska, in 1912, strongly favours a cosmic origin of the cloud particles.

An analysis of the results of the measurements of the present plates will be published elsewhere.

Department of Natural Philosophy,
University of Edinburgh.
2nd October 1950.
J. PATON.

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## High Frequency Permeability of Ferromagnetic Materials

With reference to the paper by Millership and Webster (1950), it seems desirable to mention some later work which bears on the generality and precision of the results there given.

All the results given show a monotonic decrease of the inductive permeability, $\mu_L$, with increase of frequency, although more recent experiments on nickel carried out in this department indicate that there are definite deviations from this simple monotonic form, showing that this must not be regarded as general. It is hoped that these results will be published when completed.

In connection with the precision it should be stated that each point on the graphs in the above-mentioned paper represents the mean of a group of experimental values. Redetermination of the inductive permeability for annealed iron, using improved apparatus which gives a much reduced scatter of the individual experimental points, has shown a somewhat more rapid decrease in the 150 to 250 Mc/s. range than is shown in Figure 5 of the paper. The general conclusions, however, remain unaffected by the more precise results obtained in this new determination.

Physics Department,
The University, Leeds, 2.
1st November 1950.
F. V. WEBSTER.
K. S. DRIVER.

MILLERSHIP, R., and WEBSTER, F. V., 1950, *Proc. Phys. Soc.* B, **63**, 783.

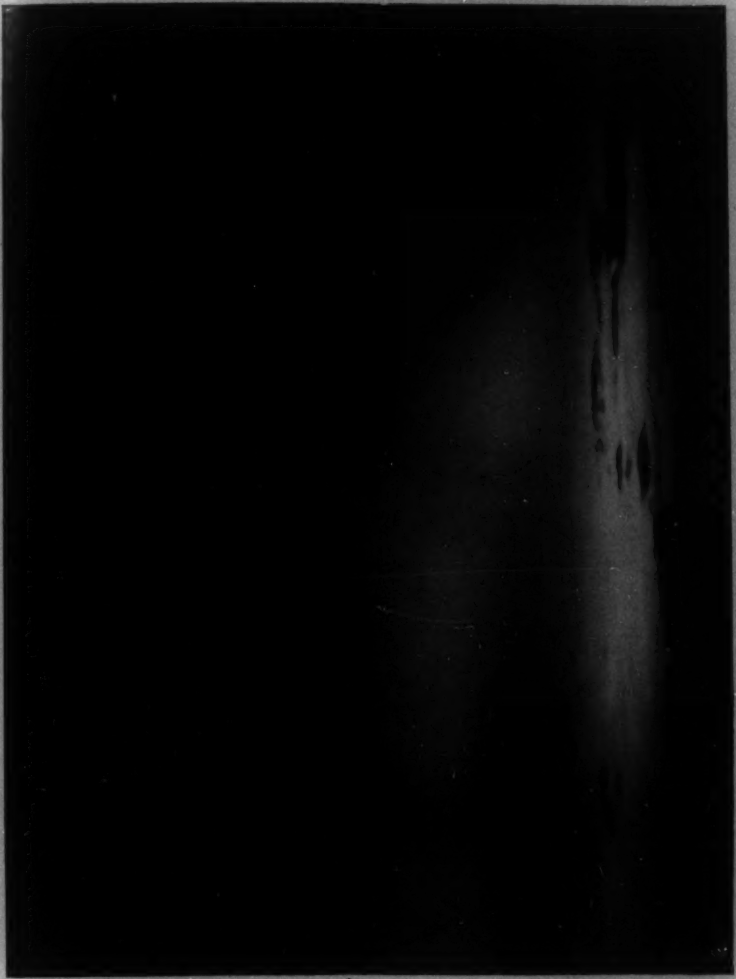

PLATE I.

Auroral arc with sunlit ray and luminous night clouds just above the horizon.
Photographed at Abernethy, Perthshire, 2355 G.M.T., 24th July 1950.

PLATE II.

The luminous night clouds at 0148 G.M.T. on 25th July 1950, showing characteristic wave structure.

# Obituary Notices

## JAMES REGINALD ASHWORTH

THE death of James Reginald Ashworth on 9th July 1950 deprived scientific circles in the north-west and farther afield of a highly esteemed member. At the age of eighty-nine, he had only recently resigned from the office of honorary secretary of the Rochdale Literary and Scientific Society after sixty-five years, and he had been a co-opted member of the Public Libraries, Art Gallery and Museum Committee of Rochdale for sixty-three years.

Dr. Ashworth built up a considerable reputation in the wider scientific world. He wrote eighteen books and pamphlets dealing mainly with various aspects of magnetism. In May 1939 he was invited to address a conference on magnetism organized at Strasbourg by the French Central Council of Scientific Research.

For many years he had been deeply interested in the subject of atmospheric pollution and its effect on the transmission of ultra-violet radiation. He installed a gauge on the roof of the Rochdale Technical School and made daily observations to the Smoke Abatement Society.

At the time of his death Dr. Ashworth was curator of the Manchester Literary and Philosophical Society. Up to the end of last year he was in good health, attending the Council meetings regularly and giving valuable advice.

He had been a member of the Physical Society for fifty-three years and was also a Fellow of the Royal Meteorological Society.

The following tribute was paid to Dr. Ashworth by the vicar of Rochdale (the Rev. G. E. N. Molesworth :—

> " He said he valued above all things integrity and truthfulness of heart and mind. So indeed his life has proved. We shall not see his like again and we shall be immeasurably the poorer for it. He has reached the end honourably with his finger on the pulse of passing events."

## ALFRED JOSEPH BULL

DR. ALFRED JOSEPH BULL was an outstanding personality of our time. Few men have gained distinction and honour in such widely different spheres of thought and action as geology and the graphic arts.

He started his career as an analytical chemist and later joined the staff of Messrs. Lumière where he was one of the pioneers in colour photography. He continued research on colour vision at the Northampton Institute where he lectured on Optics. In 1903 he joined the staff of the School of Photo-engraving and Lithography at the old house in Bolt Court, once a residence of Dr. Johnson. Here with A. J. Newton, the Principal, he investigated the problems of colour photography in relation to photo-engraving, especially with regard to the sensitivity of photographic emulsions, the transmission of filters, and the testing of lenses for colour work.

In 1912 Bull became Principal of the new school built on the original site in Bolt Court. He approached his new responsibilities from a new angle. Research was not regarded as a proper activity for a technical school by the Education Authority of that time. Despite this, Bull concentrated on technical research to such a degree that it became the educational tradition of Bolt Court and helped to establish its international reputation. His own contribution from the year 1912 to the time of his retirement in 1946 was considerable. His special field was colour reproduction, and his work formed a foundation for much of the development of ' colour masking ' in recent years. Alone or in collaboration with his staff he published numerous papers on other aspects of photo-engraving including tone-rendering, densitometry, and ' pattern ' in colour reproduction.

In 1933 Bull was elected President of the Royal Photographic Society and continued in office to play a leading part in the Fox Talbot centenary the following year. During the last war he was concerned with research for the War Office, and Field Survey, and on techniques for the production of ' graticules '. The story of this work and of the establishment of munitions factories at Bolt Court and Putney has been told in a report he published at the end of the war and in a series of papers read at the Royal Photographic Society.

Bull's geological work covered a wide field including geomorphology, stratigraphy and tectonics. He became a member of the Geologists' Association in 1916, served on its Council for many periods, became President, 1926–38, and subsequently served as Vice-President and successively as Secretary of the Fields Meetings and Publications Committees. As Chairman of the Weald Research Committee from its inception he was active in carrying out an important part of the geological survey of the South Downs, the river valleys and the South Coast.

He was also a Fellow of the Geological Society and for some time Treasurer of the Mineralogical Society. As a result of several visits to the Alps with Swiss geologists he became interested in tectonics and contributed valuable papers on the theory of Mountain Building. In this work he carried out many interesting laboratory experiments, and his papers were usually illustrated with photographs he had taken in the field.

His personal friends will remember Bull as a cheerful and interesting companion, always happiest when pursuing his geological interests. In retirement he continued much of his committee work and he was following his geological studies with vigour up to the time of his death. He was a modest man seeking only to find the truth in all that he did. Nothing could have pleased him more than the knowledge that the work he had started in such diverse fields would be continued in the same spirit of sincerity. H. M. C.

## FREDERICK CHARLES CLARKE

THE death of Mr. F. C. Clarke, a Fellow of the Physical Society since 1911, occurred after a very short illness on 16th February at the age of 72. His death will be deeply regretted by many old friends.

He was a native of Plymouth, having a brilliant scholastic career as Foundation Scholar of the Plymouth Grammar School. From there he proceeded with a National Scholarship to the Royal College of Science in 1898 and gained the Associateship of the College in Physics, and the Honours Degree in Physics of the London University.

Clarke was deeply interested in the growing activities of the many large and well-equipped Technical Colleges, and he acquired unique teaching experience by holding appointments at the Birkbeck College, Chelsea Polytechnic, and as teacher of Mathematics and Physics at the West Ham Technical College, founding courses which were recognized for Internal Students of the London University. In this way Clarke gained exceptional experience in studying the requirements of students both for University work and for work in Technical and Applied Physics, and thereby qualified himself for his subsequent career.

The outbreak of the war 1914 to 1918 removed him temporarily for military duties, as he was an active member of the Corps of London Electrical Engineers, R.E.(T). He was actually in camp at Dover commanding a Searchlight Company at the outbreak of hostilities, with the rank of Captain R.E.(T), and subsequently assumed command of the Dover searchlights with the rank of Major throughout the war.

After the war he resumed his duties as Lecturer in Physics at West Ham until 1930, when he was appointed as the first Principal of the Rotherham College of Technology, a position he held until 1948. His capacity was so highly appreciated that his term of service was extended five years beyond the normal retiring age. During that time Clarke devoted all his energies to equipping laboratories, recruiting staff, contacting the employers of the great steel and other industries of the neighbourhood, and the social life of the students of the College.

On his retirement many tributes were paid to the zeal with which he fulfilled his duties. The amazing growth of the College was stated as being undoubtedly due in no small way to his tremendous drive and to the intimate relations he held with his staff and students. The original staff of four increased to 42 full-time and 88 part-time teachers, and the number of students increased from 750 to over 3,000. The courses included all branches of engineering, from the principles of management and administration to the duties of foremen and supervisors in local works. Students worked for University Degrees in electrical engineering and metallurgy, chemistrgy, physics and mathematics.

Clarke kept in close touch with all developments in Technical Education, both local and national. He was past-president of the Association of Teachers in Technical Institutions, and later a member of Council of the Association of Principals of Technical

Institutions. He was also a member of Council of the Sheffield Section of the Institute of Production Engineers, and of the Sheffield Subsection of the Institution of Electrical Engineers, of which he was an Associate Member.

Probably visits to the Physical Society Exhibitions gave him the most pleasure, for here he could meet old friends and watch the wonderful achievements of those, who unlike himself, could devote themselves to physics research : he was at heart a physicist.

After a bare eighteen months of retirement he undoubtedly paid the penalty of the strain of a very active life.

He leaves a widow, a son and a daughter. W. S. T.

## JAMES ARNOLD CROWTHER

THE sudden death of Professor J. A. Crowther on 25th March at the comparatively early age of 66 came as a great shock to his many friends, and is a severe loss to the Physical Society, of which he had been a distinguished member for nearly 23 years. Since his retirement in 1946 he had lived in Cornwall, and although for some time his health had not been good his zest for life remained, and to the last he was actively interested in literary, scientific and educational matters. When I last saw him in December 1949 there was no sign of any flagging of his spirits, and in a letter which he wrote to me a few days before he died he sounded characteristically cheerful and happy. Our deepest sympathy is due to his widow and his two sons in their time of sorrow.

James Arnold Crowther was born in Sheffield on 28th August 1883, and received his early education at the Royal Grammar School. From there he entered St. John's College, Cambridge, with an open Science Scholarship, and after gaining First Class Honours in both parts of the Natural Sciences Tripos he started work as a research student in the Cavendish Laboratory under J. J. Thomson. He held the Mackinnon Studentship of the Royal Society, and in due course became a Fellow of his College. Most of the investigations going on in the Cavendish at that time were connected with ionizing radiations, and so it came about that Crowther entered the field of x-rays and radioactivity, which was to become the chief scientific interest of his life, and to which he made a number of important contributions. Later he became a pioneer in the study of the effects of penetrating radiations on living cells, and may fairly be said to have laid the physical foundations of radio-biology.

Crowther early showed exceptional gifts as a teacher. His style was lucid and always interesting, whilst his understanding of human nature, combined with a keen sense of humour, enabled him to keep his audience alert and eager to hear what was coming next. After serving an apprenticeship as a demonstrator under Dr. G. F. C. Searle, he was put in charge of the electrical laboratory, and also gave stimulating lectures on Electricity and Magnetism to students working for Part I of the Natural Sciences Tripos. When the Cambridge Diploma in Radiology was started in 1919, Crowther was appointed to the new University Lectureship in Physics as applied to Medical Radiology, a position which he held for four years, and in which he did splendid service in teaching comparatively advanced physics to medical graduates whose previous knowledge of the subject was scanty, and whose interests were mainly clinical. He was particularly well qualified to do this as, during the 1914–18 war, he had worked in the x-ray department of a war-time hospital, and had helped to develop radiographic technique at a time when all the work was done by means of gas tubes, and only the most rudimentary forms of equipment were available.

In 1924 Crowther was appointed to the Chair of Physics in the University of Reading, where he threw himself energetically into the task of organizing the physics teaching and building up a school of research. He was the author of a number of successful books, of which the best known is his *Ions, Electrons and Ionizing Radiations*. This book has passed through many editions since it first appeared in 1919, and has probably been read by every honours student of physics throughout the Empire during the past 30 years. It was no small task to keep a work of this character up to date, for the issue of a new edition had to be followed almost immediately by preparation for the next. His *Manual of Physics*, also first published in 1919, still retains its popularity, whilst the *Handbook of Industrial Radiology*, of which he was editor, has passed through two editions in five years. In his earliest work, *Molecular Physics*, published in 1914, as much as in his later writings, he showed the same clarity of style and the same gifts for arousing interest that characterized his lectures. Little more than a year ago he gave a most successful broadcast on his old Professor, J. J. Thomson, for whom he had always a profound admiration and affection. This talk brought him letters of appreciation from many old Cavendish men who shared his deep regard for 'J. J.' Crowther had a very wide experience as an examiner, and was as much interested in the proper conduct of School and Higher Certificate examinations as in special honours B.Sc. work.

Crowther served on the Council of the British Institute of Radiology for a number of years, was President in 1936–37, and gave the Silvanus Thompson Memorial Lecture in 1937. In 1947 the Institute elected him to Honorary Membership, and he was also an Honorary Member of the Faculty of Radiologists. He was a Founder Member of the Institute of Physics, of which he was Honorary Secretary from 1932–46, Vice-President in 1948, and to which he gave devoted service. He did much to raise the status of physics as a profession, and during the second World War he served for two years as Vice-Chairman of the Parliamentary and Scientific Committee, which was concerned with the proper use of scientific manpower.

Crowther's married life was a very happy one, and he owed much to the loyalty and devotion of his wife. Those who were privileged to enter his home could not fail to be struck by the warmth and friendliness of its atmosphere. His conversation was always amusing, and he had a ready wit, but he was essentially a man of simple tastes, who enjoyed a game of tennis or golf, and was always ready in the evening to make up a four at bridge. He took pleasure in foreign travel and spent many happy holidays abroad with his wife and family touring Europe in his car. More recently he had formed the wish to visit New Zealand to see his son who is engaged in academic work there. He had some gifts as a singer, and when he attended the annual Cavendish Dinner he could always be relied upon to give a tuneful and spirited rendering of the special Cavendish songs, most of which were written by the late A. A. Robb. He was particularly fond of the Gilbert and Sullivan operas, which he knew almost by heart. He had complete gramophone records of many of them, and never tired of playing them over in the evenings, particularly if he had a sympathetic and appreciative audience. His early association with the Cavendish Laboratory, as well as his later work at the Institute of Physics, brought him into intimate contact with large numbers of physicists, who will always remember him as much for his human qualities and personal charm as for his notable scientific achievements. He had a great gift for friendship, and will be sadly missed by all who knew him. G. STEAD.

## GEOFFREY E. F. FERTEL *

THE tragic death of Geoffrey Fertel on 19th January 1949 marked the end of a man of remarkable ingenuity and individuality, great integrity, and yet considerable diffidence.

He was born on 19th November 1913, and his father was organist at Bromley Parish Church. He was educated at Sevenoaks School, and later came to study physics at the Royal College of Science. Here his almost prodigious experimental ability developed to the full and, not content with the experiments provided in the degree course, he devised and carried through a great deal of practical work at his home, including even an absolute determination of the ohm and the construction of Geiger counters and the necessary auxiliary equipment. All this was done with materials and components costing at most a few shillings, obtained from the most diverse places. He was a rigid functionalist and, especially at this time, took a kind of inverted pride in the fact that the non-essential parts of his apparatus were never unnecessarily good. Although he devoted most of his energy to practical physics and took little part in games or any other social activities, his interests were by no means as narrow as one might have thought on first meeting him. One discovered after a time that he was enviably well read in literature, both prose and poetry, and something of an authority on wild plants. It was at this time too that his practical joking was at its maximum—he had several ways of producing loud explosions at unexpected times and places, and one lecturer may perhaps remember spending an embarrassing time trying to bring the slide lantern into operation, only to find at length that one of the carbons was insulated by a tightly fitting black paper tube.

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* This notice was unfortunately just too late for publication in the 1949 volume of the *Proceedings*.

After graduating he worked under Sir George Thomson on neutrons, much of his time being spent with a team working on a time-of-flight experiment. Shortly before the war he left London to take up a post at the University of Bristol, where he did some work with Professor C. F. Powell on the photographic plate technique of detecting nuclear particles, then still in an early stage of development. On the outbreak of war he joined a mobile group fitting up coastal radar stations, and shortly after went on a cruise to the Mediterranean on one of H.M. ships to carry out radar trials. On such a retiring and introspective person these turbulent experiences had a profound effect. For the rest of the war he worked in Admiralty laboratories on radar, first at Portsmouth, later at Nutbourne, and finally back at the Physics department of Bristol University, where an Admiralty team under R. W. Sutton was working on the development of new valves. He specialized in gas discharge devices for common aerial working, and it is probably in this field that his most important work was done. At the end of the war the valve group moved from Bristol, but he remained there, now back on the University staff, and spent some time teaching and applying his knowledge of radar technique to more academic problems.

He spent most week-ends exploring the countryside on foot or bicycle, and it was at Bristol that he developed an interest in caving and canoeing. He delighted in any sort of climbing, and had an alarming technique of proceeding upside down, feet first, which was particularly effective when applied to trees. He will be remembered affectionately by everyone in the Physics Department at Bristol for the Heath Robinson devices which he delighted in making for Christmas parties. One of the best of these was a cigarette-lighter of arresting proportions. Built on the stand of an old astronomical telescope, and having a large and crooked tree-trunk as a prominent part of its structure, it attained a height of seven or eight feet, and was set in operation by first winding up some heavy pieces of scrap iron to a considerable height. On pulling a lever these weights were released and drove a primitive dynamo, the output from which heated a small wire. In the meantime the cigarette, which had been inserted in an ornate holder made from an old curly brass candlestick, was brought into contact with the wire, and had applied to its other end a partial vacuum produced by means of a bicycle pump, also driven by the weights. The probability of successful operation of the lighter was about the same as that of the more conventional kind. Apart from diversions of this kind he was an excellent though unconventional mechanic, and throughout his career made most of his apparatus himself, usually with remarkable speed.

In 1948 he obtained leave of absence from Bristol University to join the team working on the cyclotron at Birmingham University, under Professor Oliphant. By this time he had lost much of his former reserve and made many new friends.

The precise nature of the accident which resulted in his electrocution can never be known, since he was by himself in the cyclotron pit at the time, but there is no doubt that he died instantly.

The amount of published work he has left hardly does justice to his creative ability. Technical discussion with him was never easy: his mind was always leaping ahead, or perhaps sideways, but always to fresh ground, and whether or not any definite conclusion had been reached at the end, there were always dozens of new ideas and original suggestions for overcoming practical difficulties. All who knew him will agree that his personality was unique, and the contribution he had to make to science, not only by his own work but by the energy and enthusiasm which he could inspire in others, can ill be spared.

D. F. GIBBS.

## CYRIL OWEN GREEN

Cyril Owen Green was born on 24th November 1920 and was educated at Campbell Square School, Northampton. He next studied as an evening student at Northampton College of Technology and after obtaining the Higher National Certificate in Electrical Engineering decided to study Physics. He entered University College, Nottingham, in October 1942 with a Foundation Scholarship. After a short course Green's war service was spent as an electrical officer in the Fleet Air Arm.

Green returned to Nottingham in 1945 and obtained first class Honours in Physics in the London B.Sc. examination in July 1948. He then became a research student at

Manchester University working on the properties of oxide-coated cathodes. Later, in 1949, he joined the cosmic-ray group directed by Professor P. M. S. Blackett. He completed a useful analysis of the problem of ionization measurements in the Wilson cloud chamber.

Last autumn Green spent three months helping to erect a large electromagnet at the observatory on the Pic-du-Midi in the Pyrenees. In May 1950 he again went to the Pyrenees in order to install a Wilson cloud-chamber in the field of the magnet. On 11th May, a colleage, and three French workers at the observatory stated to climb the Pic. They were travelling light, made good progress and were resting at 8,000 ft. when Green suddenly collapsed and died shortly afterwards. The other members of the party and the observatory staff rendered what help they could, but it was of no avail.

In this way a career of great promise was brought to an early end. Green made many friends who will remember his quiet unassuming manner and unselfish disposition In 1948 Green married Miss Lorna Rumney.

N. DAVY.

## HERMAN SHAW

THE sudden death of Dr. Herman Shaw, Director of the Science Museum, at his home at Barnes on 4th May 1950, came as a surprise and shock to his many friends in the Museums and scientific worlds and in particular to the Physical Society, of which he was Honorary Treasurer. His record as administrator, scientist and man of affairs was an impressive one, and his friendliness and accessibility had endeared him to a very wide circle of acquaintances.

He was born on 14th October 1891, the only son of the late Mr. G. H. Shaw of Huddersfield. He was educated at Bradford Grammar School, where he obtained a Governors' scholarship, and at the Imperial College of Science and Technology, which he entered as a Royal Scholar in 1911. In 1913 he won an Aeronautics Research Scholarship at this College. In the first World War he became a Lieutenant in the R.N.V.R. in 1915, a Flight Lieutenant in the R.N.A.S. in 1916, and a Captain in the R.A.F. in 1917.

Shaw entered the Science Museum in 1920, and for the remaining 30 years of his life his energies were devoted to the development and expansion of its collections and services, and to research in Applied Geophysics, in which subject he became a world authority.

His work in Geophysics began in 1921, when he and the late E. Lancaster-Jones obtained permission to make extensive laboratory tests on an Eötvös torsion balance which had just been acquired by the Museum. The tests were described in two papers published in the Physical Society's *Proceedings*, and proved so successful that they were soon extended to field experiments. These were carried out first at a site in Shropshire and then in 1925 at Hodbarrow, Cumberland, using a British-made torsion balance in the design of which Shaw and Lancaster-Jones played a leading part. The experiments, the first of their kind to be carried out in the British Isles, showed in a striking way that an underground rock structure—in this case a limestone ' dome '—could be located and delineated with the aid of the torsion balance.

In 1928 Shaw and Lancaster-Jones designed their ' gravity gradiometer ', a form of torsion balance specially modified for field work in the measurement of gravity gradients, and much smaller, lighter and quicker in operation than the earlier Eötvös form of balance.

Shaw's interest in Applied Geophysics was by no means confined to the gravity method of prospecting, but soon grew to cover all its aspects, and he became recognized as one of the leading authorities on the subject, on which he published in all about 20 papers.

Concurrently with his geophysical work, most of which was carried out in his spare time, Shaw was very active in purely Museum work. The ten years following World War I was a busy period for the Museum, for they saw the completion and occupation of the new East Block, which was formally opened by H.M. King George V in 1928. Considerable expansion took place in the Science Collections, and in this development Shaw played his full part. He was also responsible to a large extent for the special exhibition on Applied Geophysics held at the Museum in 1931, and was co-author of the Museum Handbook on the subject published at this time.

He was made a Deputy Keeper in 1931 and in 1935 was promoted Keeper of the Department of Physics and Geophysics. In 1939 he was put in charge of A.R.P. arrangements at the Museum, and later took full charge of the arrangements for the evacuation to the country of the greater portion of the Collections. This work he carried out with great efficiency and thoroughness, and it was largely due to him that the Collections came through the Second World War with a minimum of damage or deterioration.

Soon after the outbreak of the War, the Director of the Museum, Col. E. E. B. Mackintosh, was called away for military duties, and in 1940 Shaw was made Acting Director. Col. Mackintosh later returned to the Museum, but on his retirement in November 1945, Shaw was appointed to succeed him as Director.

He resolved at once that the Museum should be opened again to the public at the earliest possible date, and applied himself energetically to the task. Partial re-opening took place as early as February 1946, and Shaw's policy received immediate justification by the fact that by the end of that year over 1¼ million visitors had already seen the limited collections then on view. Over the succeeding four years steady progress was made in rehabilitation, and at the time of his death the work was nearing completion, although somewhat restricted in total extent by the loss of the Old Building through age and war damage.

In spite of the burden imposed by this work, Shaw found time to play a notable part in many outside activities. His association with the Physical Society in particular was a long and intimate one. He was elected a Fellow in 1914, and was a member of Council from 1935 to 1939 and from 1941 to 1946, in which year he was elected Honorary Treasurer, a position which he held at the time of his death. He was also a member of the Colour and the Optical Groups. He was a Founder Fellow of the Institute of Physics.

At the time of his death Shaw was also President of the Museums Association, a trustee of the Imperial War Museum, a governor of the Imperial College of Science and Technology and a manager of the Royal Institution.

Shaw married, in 1919, Constance, the daughter of Mr. F. Shaw of Harrow. They had one son, who died as a young man in 1946, after an illness involving much suffering near its close. The loss was a severe blow to Shaw and his wife, but he bore it with fortitude, and only those in close contact with him were able occasionally to realize what it had cost him.

Amid such an active life, Shaw left himself little time for hobbies. He was, however, proud of his garden, and took considerable pleasure in motoring.

As a man, his most notable characteristics were his energy and approachability. He never spared himself in work, but obviously enjoyed the wide range of contacts which it brought him. As Director he was just, firm and kindly, and showed a practical interest in the well-being of all his staff. His sudden death at the height of his powers is a severe blow to the Museum to which he devoted so much of his energy and talents, and will be widely regretted among his large circle of acquaintances in many walks of life.

F. A. B. W.

## GEORGE WILLIAM TODD

George W. Todd, Professor of Experimental Physics in King's College, Newcastle-upon-Tyne, died on 24th February 1950, a few weeks after an operation. His health had been causing his friends anxiety for many months, but he had carried on with his work, although with growing difficulty, until the end of the previous term.

He was born in Birmingham in 1886 and, after attending a secondary school in that city, entered the University in 1904, where he studied under Poynting and graduated with Honours in 1907. After two years' research at Birmingham he was elected to an 1851 Exhibition, which he held at the Cavendish Laboratory under Sir J. J. Thomson. This was extended beyond the normal period of two years until 1912, when he took up a science mastership at the Wattville Road School in Birmingham. In the following year he was awarded the D.Sc., and was appointed to the Royal Grammar School, Newcastle-upon-Tyne, as Physics Master. Here he remained until 1916, when he joined the Munitions Inventions Department of the Ministry of Munitions, later becoming head of the Physics Section. At the end of the war he was appointed to the newly established Chair of Experimental Physics at Armstrong College, Newcastle, where he remained until his death.

His research work was mainly experimental and covered a wide field. It was characterized throughout by elegance, accuracy and thoroughness, for he possessed in full measure the true physicist's appreciation of the aesthetic aspect of his subject. These qualities were first shown in his 1909 paper on the thermal conductivity of air and other gases, and they were equally apparent in his 1928 paper on an expansion method of measuring the Peltier coefficient. His sojourn at the Cavendish Laboratory led to the publication of four papers on ionic mobilities, and provided him with a fund of pleasant reminiscence from which both he and his friends derived much enjoyment. It was evidently the most stimulating and satisfying period of his scientific career.

His scientific work during the 1914–18 war was on quite different lines, being mainly concerned with gas reaction velocities, and on this subject he published in all some six papers. The most important of these was probably that in which, in collaboration with S. P. Owen, he derived expressions for the temperature coefficients of the velocity and equilibrium constants of homogeneous gas reactions. A little later he showed that physical equilibrium could be treated in a similar manner, and so obtained a vapour pressure equation which gave good agreement with experiment over a wide range of temperature. In the same year (1920) he published the results of some work on the rate of flow of gases through capillary tubes, from which he attempted, with a considerable measure of success, to establish a formula which should be applicable down to the lowest pressures.

At the time of his appointment to Armstrong College, and for some years later, research facilities in the Physics Department were very meagre and the teaching duties heavy. He published several further papers, and prepared a new and revised edition of Poynting and Thomson's *Properties of Matter*, but devoted himself mainly to lecturing, both inside and outside the College. In this he was most successful, having the gift of clear and simple exposition and a sympathetic appreciation of the difficulties of even the least intelligent of his audience. These attributes, together with his kindly and genial attitude to his students, earned him their esteem and affection, and his death was keenly felt by them. His colleagues also held him in the highest regard, for his sincerity, good-nature and unselfishness were patent to all. During the second war, when half the staff, including myself, were away, he took charge of the department and shouldered a very heavy burden of teaching and administrative work for nearly six years. This was entirely typical of him. He never sought responsibility, but was always ready to take on extra work for the benefit of others.

His main interests outside physics were gardening and painting. He was for many years a member of the Newcastle Society of Artists, and for the last few years of his life its Chairman. He was a Founder Fellow of the Institute of Physics, a member of the Faraday Society, a Fellow of the Physical Society since 1922, and Editor of the Durham University Philosophical Society's *Proceedings* since 1921. He gave numerous extra-mural courses and lectures, which were invariably well attended, and served as a member of the Investigating Panel for Higher Certificate Examinations in 1924–6, and again in 1937–8.

He will be remembered not only as a physicist who made many and diverse contributions to knowledge and as an unusually gifted expositor of his subject, but also as a man of most kindly and lovable nature. He will be sadly missed by his innumerable friends.

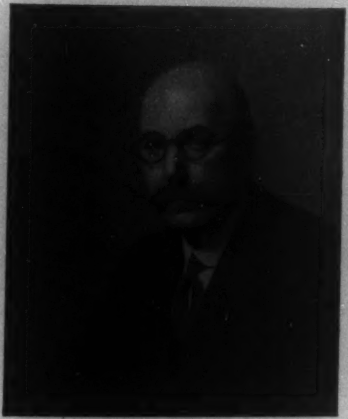
W. E. CURTIS.

## ALEXANDER WOOD

The late Dr. Alexander Wood, M.A., D.Sc., Fellow and Tutor of Emmanuel College and Lecturer in Experimental Physics in the University of Cambridge: "I have no specialist's contribution to make. My rôle has always been that of interpreter rather than the research worker, and I only take courage from the fact that never was the rôle of interpreter more essential than in our own time." Thus Wood spoke of himself in his inaugural address to the Acoustics Group of the Physical Society (March 1947). He modestly left it there. But there is something more to be said.

Why with his intelligence and force was he, in these times, an academic scientist only. He was born in 1879 and died in 1950. He came to Cambridge from Lord Kelvin's degree lectures at Glasgow University just at the right time. He had the passion for enquiry. He was vitally interested in the scientific arena and was historian of the Cavendish Laboratory; yet in research had no contribution to make. I believe he deliberately left the laboratory, that he left it because, though enquiry was great, there was in fact something greater, and that his significance lies in the fact that in the considerable stress of our times he would not

HERMAN SHAW



JOSEPH ALFRED BULL

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JAMES ARNOLD CROWTHER

give up either orthodox Christianity on the one hand nor modern science on the other. Of course it is not surprising that he was also religious. In all periods there have been acute scientists who have seen nothing contrary to good sense in associating a Creation with a Creator and who would turn from sense data *in simplici intuitu*. But with Wood there was (in my opinion) a distinguishable factor. He was of the blood and the temper of the Scottish Reformers. He had the instinct of the theologian to organize the whole of knowledge, and felt that one half—Creation—is ultimately unintelligible without the other half. For him Truth was the aim and the battlefield, but Truth meant ' the Whole '. And it is natural that in his own historical period the battlefield must include the physical sciences because in that direction, in response to a profound gaze of man's faculties (what the mystics call a *quality of attention*), new knowledge and new power was emerging. He well knew the excitement of the chase—when detective plus inductive powers are in full cry—and always insisted that the creative faculties were as fully involved. But where was it leading, and into what theological pattern could it fit ? Once early in the thirties we discussed this in the Fellows' Garden at Emmanuel. The debate in Christian terms went somewhat thus : " Autonomous enquiry is inevitable—to know is a passion. But God permits scientists, endows them with infinite curiosity; then he permits ultimate risks, certainly as part of the contingency of free-will and a cosmic design not fool-proof. What then becomes of the Kingdom of God on earth? Here is the dialectic of Christianity: on the one hand the City of God, the reformer's inspiration, on the other the eschatology of the New Testament, the facing of catastrophe; religion is the technique of being at home on the edge of the precipice. It is for this reason that Christianity is so distasteful to Reason but so true as an account of Life itself." But Wood did not agree. When the deeps opened under him in the course of argument he had a peculiar metaphysical smile known to his friends—a smile of exhilaration. No, Christian Truth must include cosmic Truth. He meditated much on the Gospels and saw Our Lord before Pilate—" For this cause came I into the world, that I should bear witness to the truth ". Wood writes: " The induction upon which we are called upon to act is not merely that the good life is the life lived and taught by Jesus: it is the whole view of God and the world from which his life issued and in virtue of which the life was and is possible." In his later years as nuclear physics developed, the conviction of an all-in Christian Truth was not easy. Deeply anxious, he would look out over his peaceful Broads, or scan the rows of new Cambridge houses which, as councillor and chairman, he had done so much to build, and the words ' radioactive mud ' would frame themselves. Yet he did not despair. There was always something to be done in the immediate battlefield. He could preach, argue, defend his religion, " but first he followed it himself ". His action came out of his inner life. So he allowed himself to be many-sided, yielding to the service of God now here, now there. Like many pacifists, he was a bonny fighter, and in committee was a master of the tactical battle. The range of his activities was astonishing. In addition to his duties as fellow and tutor of his college, and (later) as chairman of the Borough Housing Committee, there was also the Student Christian Union (he was a frequent speaker at Swanwick), the Peace Pledge Union, the United Nations Food Organisation, the Cambridge Divisional Labour Party, the Welfare of Youth Union, the Fellowship of Reconciliation, and many others besides.

But action was not merely an anodyne. It was the testing of theory: it was part of his experimental method. Science was always much more than enquiry. It was also the selecting of what to enquire into, and it was the organizing and unifying of our knowledge. " We are face to face with an amazing multiplicity and diversity of phenomena constituting our physical environment. If we are to live intelligently we must organize our past experience so as to make it readily available for meeting the circumstances of the moment and for providing for the needs of the future. In the earlier stages the results of past experience are accumulated instinctively as an unorganized background in our minds. Later we became conscious of the need for organization, and Science is the response to this very practical need." He then continues: ". . . . in a sense religion is also a response to a similar practical need. Man's experience is not confined to sense data. He has to live in a world which, on the face of it at least, is a moral and even a spiritual world as well. Just as successful living in the first of these worlds—or the first plane in the only world we know—involves a mastery of our material environment, so successful living in the world's wider aspects means mastery in the moral and spiritual realms also. There is no more reason for us to be the playthings of our moral environment than there is for us to be the sport of our physical environment." The test of both, then, is successful living. "We want to understand life. We want to discover the great induction which will relate and give meaning to all its complex and puzzling experiences." He points to the fact that in Science it is not, at any particular time, the fashionable 'verification' which truly convinces us of a theory; it is the fact that the theory over a long period has contributed to render phenomena intelligible. And if it does so all kinds of anomalies and contradictions are endured. "Newton's Law of Gravitation has been subjected to innumerable verifications. Yet it seems probable that gravitation is a property of space, and not, as Newton thought, of matter." The importance of 'verification' is, however, that it shall make intelligible our future experience also. "This also we may fairly demand of religion. Our lives ought to be a continuous verification. We must not only understand the situations which have arisen in our past experience; we must be equipped to meet new situations as they arise. This double criterion is undoubtedly our test of scientific truth, and I conceive it to be the *intellectual* test of religious truth as well." (Italics are his.)

Wood thus gives philosophical significance to 'successful action' in the total issue of theoretical knowledge, and he transfers it from Science to Religion. Hence the importance of his active life. He was, moreover, fully conscious of the metaphysical weakness of parallels drawn from sense data. He believed there was no such thing as a logical refutation of philosophical idealism. Our conviction of the reality of things arises from constantly living among them and acting as if they were real. But if we direct our attention to, if we gaze at, material things only, their sense of reality grows because our powers of concentration are almost demonic in their summoning power. But by the same test spiritual things ignored lose their sense of reality. "In a world where the things of sense and time press on us more continuously and more aggressively than ever before, the reality of the unseen can only remain an unshakable conviction if we allow it to make its own impact upon us, and that means a scrupulous anxiety to preserve times and seasons when alone, or in fellowship, we may have leisure for worship and meditation and prayer. All these times must be opportunities for the unseen to make its own impact, and not times devoted to a fussy activity of our own. They must include spaces of silence in which God can speak to us. As we enter the doorway of religion it is simplicity that matters most. If we happen to be scientists or theologians we must not be disloyal to the spirit and temper and results of our scholarship, but neither must be wait outside in the vain hope that all our difficulties will be resolved."

The bulk of the quotations given above are taken from his book *In Pursuit of Truth—A Comparative Study in Science and Religion* (S.C.M. Press, 1927) and from *Some Implications of Modern Physics* (Independent Press Ltd., 1936).

H. BAGENAL.

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# ABSTRACTS FOR SECTION A

## *Film Transfer in Helium II: I—The Thermo-Mechanical effect*, by J. G. Daunt and K. Mendelssohn.

*ABSTRACT*. The existence of a thermo-mechanical pressure difference has been established with the transfer film instead of a capillary connecting the two volumes of liquid helium II. It is shown that under these circumstances reversible conditions are closely approximated since return flow of heat or liquid through the film is negligible. The speed with which a thermo-mechanical pressure difference can be established is limited by the same critical transfer rate as that determining film transport under gravitation.

## *Film Transfer in Helium II: II—Influence of Geometrical Form and Temperature Gradient*, by J. B. Brown and K. Mendelssohn.

*ABSTRACT*. Experiments have been carried out in order to investigate a possible dependence of the film transfer in liquid helium II on the geometrical conditions of the measuring arrangement or on a temperature gradient along the film. Using transfer vessels of different shape, no variation in the rate of transfer could be detected. Equally no change from the ordinary rate of flow could be found under temperature differences up to $3{\cdot}5\times10^{-2}$ deg.

### *Film Transfer in Helium II: III—Influence of Radiation and Impurities*, by R. Bowers and K. Mendelssohn.

*ABSTRACT*. In contradiction to the results of de Haas and van den Berg, the complete exclusion of thermal radiation has not been found to increase the transfer rate beyond the normal value. It was observed, however, that contamination of the transfer surface with condensed gases will cause a greatly increased film flow. The effect of impurities on the transfer has been studied in detail and the conclusion has been reached that the high and variable transfer values recently reported by some observers are to be ascribed to this cause. The bearing of the effect on cryogenic technique is discussed.

### *Film Transfer in Helium II: IV—The Transfer Rate on Glass and Metals*, by K. Mendelssohn and G. K. White.

*ABSTRACT*. The rate of transfer $R$ has been measured as a function of temperature along surfaces of glass, platinum and nickel. While the $(R, T)$ curves have all approximately the same shape, the absolute values of $R$ on metals were found to be much higher than on glass. The effects of polishing and degassing of the transfer surfaces have been studied. Possible explanations for the observed effects are discussed.

### *The Disintegration of the Deuteron by Neutron Impact*, by B. H. Bransden and E. H. S. Burhop.

*ABSTRACT*. The method of distorted waves is applied consistently to the calculation of the cross section for disintegration of deuterons by neutrons. At an incident neutron energy of 11·5 Mev. the cross section calculated for the process ($6{\cdot}4\times10^{-25}$ cm$^2$) is about one-third that obtained in previous calculations but is nevertheless an order of magnitude greater than that indicated by the rather meagre experimental data. The calculated cross section is also larger than the maximum permitted by considerations of the conservation laws. This indicates that the n–d interaction is so strong that the requirements for the validity of the distorted wave method are not satisfied.

At an incident neutron energy of 5·1 Mev. where it is considerably less than the permitted maximum, the calculated cross section is of the same order of magnitude as that deduced indirectly from measurements of the p–d disintegration cross section, allowing for the effects of the electrostatic repulsion between the proton and the deuteron.

### *A Coincidence Absorption Study of the Decay of* $^{181}$*Hf*, by E. W. Fuller.

*ABSTRACT*. The decay of 46 day $^{181}$Hf has been investigated by absorption coincidence techniques using an integral type delayed coincidence recorder, and a special counter with which the absorption curves of low energy electrons may be obtained.

The results confirm a previously published scheme for the main mode of decay, namely that a 400 kev. $\beta$-ray from $^{181}$Hf leads directly to the 20 $\mu$sec. metastable state in $^{181}$Ta. This decays by emission of a 130 kev. $\gamma$-ray, followed by a 470 kev. transition to the ground state. Instantaneous coincidences were found to involve the $\gamma$-transitions in the main branch, electrons from $\gamma$-rays of 87, 133 and 340 kev., and a $\beta$-ray of approximately 440 kev.

It is suggested that about 30% of the $^{181}$Hf nuclei in the 46 day state decay by a highly forbidden $\gamma$-transition (133 kev.) with an 87 kev. $\gamma$-ray in cascade to the ground state. This decays through a 440 kev. $\beta$-transition to an excited state (340 kev.) of $^{181}$Ta. Half life and conversion coefficient values enable spins and relative parities to be assigned to each of the levels occurring in the decay scheme proposed.

### *On the Development of the Nucleon Component of the Cosmic Radiation in Air*, by H. Messel and D. M. Ritson.

*ABSTRACT*. The development of the nucleon component in air is considered in terms of a model which is an extension of that proposed by Heitler and Jánossy. The numerical results obtained are compared with experiment, and good agreement is found.

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